Measurement of very low frequency vibrations

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ABSTRACT

We have developed a number of instruments that are capable of measuring vibrations at very low frequencies (below 1 Hz). Two of these instruments are a laser strainmeter, which can respond to strains with frequencies ranging from 0 to about 100 Hz and a borehole tiltmeter which can respond to deflections of the vertical or horizontal accelerations in the frequency range from 0 to about 1 Hz. We have also modified commercial gravity meters (vertical accelerometers) and have used them to make measurements at comparable frequencies.

Most of our work has been motivated by geophysical considerations including the dynamics of the earth in seismically active regions, but the instrumentation might also be useful in vibration monitoring for other purposes.

Key Words: Low Frequency Vibrations, Laser Strainmeter, Borehole Tiltmeter

1. INTRODUCTION

Measurements of the very long period motions of the surface of the earth are useful in a number of different investigations. At the longest periods (months to years), these changes are important in the study of general plate tectonics and the dynamics of seismic zones. The earth tides are the largest signal at shorter diurnal and semi-diurnal periods. (There are longer-period components to the tides, too, but they are generally too small to measure at most sites with current instrumentation). The tides are useful in studies of earth structure because they represent the response of the earth to a known excitation[1]. Measurements of the amplitudes and phases of the tides can be used to constrain models of the elastic parameters of a region and are especially sensitive to the spatial inhomogeneities that are often found near fault zones. The very shortest period signals, with periods ranging from hours to seconds, are interesting in seismology because much of the energy released during an earthquake results in teleseismic signals with these periods.

There are many kinds of instruments that are adequate for observations of short-period seismic signals. The signal periods are on the order of seconds or minutes and these periods are sufficiently short (relatively speaking) so that considerations of the long-term stability of the instrument or of its sensitivity to slowly changing extraneous effects (such as ambient

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temperature) are usually not critical to the success of the design. On the other hand, the secular stability and the residual sensitivity to extraneous effects such as ambient temperature usually completely dominate the noise budget of instruments designed to observe signals with periods of a few hours or longer. This residual sensitivity to fluctuations in ambient temperature or barometric pressure often sets a lower-bound on the minimum detectable vibration level which is independent of the inherent resolution of the sensor or the amplification of its read-out system.

We can reduce the response of the instrument to temperature or other extraneous effects in two ways: (1) we can use a reference length that is derived from a temperature-independent standard (such as an atomic wavelength), or (2) we can balance the responses of various parts of the instrument so that the overall thermal sensitivity is zero in first order. The first technique is obviously better in principle, but devices based on an atomic wavelength standard are usually complex and too expensive to be deployed in large numbers. Devices designed using the second strategy often represent a better compromise between complexity and secular stability although they are usually inadequate to unambiguously measure motion at the longest periods. There are three reasons for this: (1) the balance among the components is unlikely to be perfect, (2) the balance is likely to degrade with time as the components age differently and (3) a perfectly balanced system will still be sensitive to temperature gradients.

In this paper we will describe three instruments which are designed using the principles we have mentioned. We have used these instruments in various geophysical investigations which are more fully described in the papers cited in the references. The focus of this paper is on the underlying design principles of the instruments themselves rather than on a discussion of the observations or their geophysical significance.

2. A LASER STRAINMETER

The earth is subjected to fluctuating stresses, and it responds to these forces by deforming in all three dimensions. If we think of a plane tangent to the earth at the point of observation, then the x and y axes are located in the plane and the z axis is along the direction of the local vertical. The relationship between an applied stress and the resulting strain is generally assumed to be linear, but while the forces and displacements are both vectors, the relationship between them is more complicated. Since a force in a given direction may produce strains in all three dimensions, the strain field usually must be described using a 3 X 3 tensor, where the i,j-th component represents the deformation along the axis i resulting from a stress along axis j. A tensor of this type has 9 independent components in general, but assuming symmetry between the x and y components (which are located in the tangent plane of the earth) and using the fact that the surface is assumed to be stress-free reduces these 9 components to 3 independent values. These three parameters can be expressed either as ϵ_{xx}, ϵ_{yy} and ϵ_{xy} or they can be thought of as the responses along two orthogonal principal axes in the tangent plane plus a third parameter giving the azimuth of these axes with respect to the local meridian. (These two principal axes may not be oriented along the North-South and East-West directions in general. The third principal axis is, of course, perpendicular to the tangent plane at the point of observation.) The complete strain tensor can therefore be derived from three non-collinear measurements of strain made in the plane tangent to the surface of the earth at the point of observation -- the strain perpendicular to this plane is assumed to be zero since the surface of the earth is assumed to be stress-free in the vertical direction.

A strainmeter measures fractional changes in the length of a baseline, and a single instrument therefore measures one component of the local strain tensor. (To be more precise, the instrument reports the spatial average of the strain along the baseline of the measurement. This averaging may result in significant differences between data from parallel instruments of very different lengths.) Conventional strainmeters measure these changes by comparing the baseline to a reference length such as a fused-silica rod or fiber. Typical secular strain-rates are 10^{-6}/year or less even in seismically active areas. The secular stability of the material and its response to changes in the ambient temperature (and possibly relative humidity) are therefore very important and usually limit the sensitivity of the instrument. In many situations, a long-period mechanical strainmeter is really just an expensive thermometer or hygrometer[2].

The mechanical stability of the instrument is still a problem even at much shorter periods down to a few hours. The thermo-elastic fluctuations are smaller, of course, but the fluctuations in the earth are smaller too and the effective signal/noise ratio of the instrument usually does not improve until the very shortest periods (on the order of seconds) are reached.

A laser wavelength provides the reference length in a laser strainmeter and the baseline is measured using this wavelength to illuminate an interferometer. Depending on the details of the design, some interferometers only provide information on the length modulo the measurement wavelength -- the integer number of wavelengths in the path and hence its absolute length are not observable. This is usually not a serious limitation since vibration measurements are usually interested only in the changes of the baseline and not in its absolute length. Although the resolution of such an instrument is derived from the wavelength of light and is therefore very high, the stability of such an instrument is generally no better than that of a conventional strainmeter unless some means are used to stabilize the wavelength of the laser.

The wavelength of a laser is defined in the large by the physics of the gain medium, but the thermal motions of the radiating atoms result in a significant broadening of the gain profile due to the Doppler effect. Within this Doppler-broadened gain profile, the oscillation frequency is determined primarily by the modes of the laser cavity, and the stabilities of these wavelengths are limited mostly by the mechanical stability of the resonator cavity itself. The wavelength of an ordinary laser therefore shows the same order of sensitivity to ambient temperature as a conventional mechanical device operating in the same environment.

There are a number of ways of stabilizing the wavelength of a laser so that it becomes much less sensitive to changes in its operating environment. The method we have chosen is called saturated absorption[3], in which the wavelength of the laser is stabilized using a servo that dynamically adjusts the output wavelength to maximize the absorption of the laser light in another material that is located inside of the laser cavity. (The absorption is termed "saturated" because at line center the laser light completely saturates the absorber so that the output power increases. This saturation occurs only at line-center because only at that frequency do the two running waves in the laser cavity compete for the same group of atoms which have essentially 0 velocity along the axis of the beam.) This method exploits a coincidence between the emission wavelength of the laser and an absorption in the reference material. A number of such coincidences are known; we have chosen to use the coincidence between a helium-neon laser oscillating at 3.39 μm and a cell of methane. The wavelength of the resulting laser shows fractional fluctuations of 10^{-11} or less and is independent of the ambient temperature, pressure or humidity at that level.

We can use this reference wavelength to measure the changes in the length of an interferometer that is coupled to the earth; the resulting instrument is a single-component strainmeter[4]. We have chosen to use a Fabry-Perot interferometer and to measure the changes in its length using a second laser that is locked to a single interferometer fringe. This method is more sensitive than a fringe-counting Michelson interferometer of the same length, but the increased sensitivity is purchased at the price of a second laser and increased complexity in the ancillary electronics.

3. STRAIN MEASUREMENTS

The ability of a strainmeter to detect geophysically significant changes in the local strain tensor is limited by the seemingly random fluctuations in strain that are recorded by all instruments[5]. Although this noise could come from either the instruments or the earth, our previous investigations have shown that well-designed laser strainmeters of widely different design and operating in widely different environments nevertheless record strain fluctuations whose power spectra are remarkably alike. We have therefore hypothesized that these instruments are all reporting the background strain fluctuations in the "quiet" earth.

Our analysis[6] was divided into three bands based on the period of the strain fluctuations: (1) very high frequency signals above 1 Hz, (2) intermediate period signals with periods from about 1 day to 1 second, and (3) long-period signals with periods longer than 1 day. The technical details of the analyses in the three bands differ in detail, but the overall conclusions do not. Apart from coherent excitations (such as the earth tides), we estimate that the background noise power spectrum across all three bands is well characterized as increasing as $1/f^2$ from 100 Hz to about 1 cycle/decade, with the power at 100 Hz being about 8 X 10^{-28} $(\Delta L/L)^2$/Hz. Apart from the coherent signals due to the tides at diurnal and semi-diurnal periods and the broad micro-seismic peak at periods of about 6 seconds, the spectrum of the quiet earth is featureless. The stability of the instrument begins to become important at a period of about a year and the advantages of the methane-stabilized laser over a simpler cavity-stabilized design begin to become apparent at that point[6].

These measurements illustrate the capabilities of the instrument and also provide a rough estimate of the background strain noise at any site. Using the fact that strain is a spatial derivative of the displacement field, it is possible to estimate the noise that would be observed by other types of instruments such as those that are sensitive to vertical velocity or acceleration. These estimates are only a rough guide, of course, since the linkage between horizontal and vertical displacements depends on the details of the noise process and on its wave-number spectrum.

Although vertical and horizontal accelerations are often equally interesting from the geophysical point of view, they may have very different effects on buildings or other structures. Horizontal strains and accelerations may induce large shear forces in tall structures, and stabilizing a structure against large shears may be considerably more difficult than stabilizing it against vertical accelerations of comparable magnitude.

4. A BOREHOLE TILTMETER

A tiltmeter measures the changes in the angle between a vector anchored to the earth and a reference direction which is usually derived from the local vertical[2]. Vertical and horizontal tilts are the angular deflections of vectors that are initially vertical and horizontal, respectively. The two tilts are the same near a horizontal free surface, but they may be appreciably different in an inhomogeneous or anisotropic environment. The difference can be quite large even in a layered medium where the inhomogeneity is confined to variations in the vertical direction only. The tilt vector has two independent components so that it can be completely specified at any point by measuring its projections along two orthogonal azimuths.

Since a tiltmeter usually uses the local vertical as its reference direction, it cannot distinguish between a mechanical change in angle (which is the every-day definition of tilt), and a horizontal acceleration of the instrument, which looks like a change in the direction of the local vertical. If the displacement amplitude is held fixed, the horizontal acceleration increases as the square of the vibration frequency so that the response to accelerations will increase as the reciprocal of the period.

Tectonic tilt rates are typically on the order of 0.2 μrad/yr and the amplitude of a typical component of the earth tides is about 0.04 μrad. To be useful in geophysical investigations, the stability and resolution of an instrument must be substantially better than both of these values. A useful instrument should therefore have a resolution on the order of nano-radians and a long-term drift of 0.1 μrad/yr or less[7]. We have constructed devices that can resolve tilts on the order of 0.002 μrad, but it is very difficult to construct an instrument whose stability in a typical field environment is better than about 0.5 μrad/yr. Most tilt measurements are therefore limited by earth noise only at short periods up to perhaps periods of a few weeks. The secular instability of the instrument dominates the response at longer periods. This limitation is only important in tectonic studies or in observations of the dynamics of a seismic zone; it is not an important limitation for vibration measurements which are usually concerned with relatively short-period effects.

A number of different sensors can be used to measure tilts[2]; some groups have used liquid-based devices similar to an ordinary carpenter's level; others have used longer-baseline liquid levels using either two vessels connected by a closed pipe or a trough half-filled with liquid and open to the atmosphere along its entire length. We have used both horizontal and vertical mechanical pendulums. Vertical pendulums are relatively simple to construct. The bob is constrained to move in only one dimension, and the motion is usually detected optically or electrically. The main problem is that they provide only a small displacement in response to a tilt so that the electrical amplification must be very high. Horizontal pendulums provide mechanical amplification and can therefore be made more sensitive for a given physical size and electronic amplification. These devices consist of a mass supported from an axis that makes a small angle with respect to the vertical (see fig. 1b). As with a vertical pendulum, the mass is constrained to move only in one direction. We mount two identical pendulums on a single plate with their sensitive axes orthogonal so that both components of the tilt vector can be observed simultaneously.

We measure the angular displacement of the pendulum using a capacitance bridge to measure the linear displacement of the plate at its free end[8]. The end-plate has an area of about 100 mm^2 and is free to move between two outer plates that are separated by about 2 mm. The capacitance between the center plate and each of the outer plates forms one-half of an a.c. bridge; the other half is formed by the center-tapped secondary of a precision transformer. When a 10-kHz a.c. signal is applied to the primary of the transformer, the voltage between the pendulum and the transformer center-tap has an amplitude that is proportional to the mechanical offset of the pendulum from the center of the two outer plates and a phase that is determined by the direction of the offset. The pendulum is initially centered between the two outer plates using motor-driven screws that level the mounting platform. Tilts or horizontal accelerations change this offset and induce voltages on the center plate; these voltages are amplified and detected using a synchronous detector tuned to the voltage applied to the bridge circuit. We calibrate the sensitivity of the device using a table which can be tilted by known amounts using a precision screw-and-lever arrangement; a typical overall sensitivity is 2 V/μrad and the full-scale range is about $\pm$ 5 μrad.

The sensor has a residual sensitivity to both barometric pressure fluctuations and to changes in temperature. The pendulums are enclosed in a sealed compartment to prevent air currents from affecting them. While this reduces high-frequency acoustic-type noise, the dimensions of the compartment are slightly changed by changes in the ambient barometric pressure; the importance of this effect is minimized by isolating the sensors from the walls of the case using the pedestal mounting arrangement shown in fig. 1a. Gradients in the ambient temperature and small differences in the coefficients of thermal expansion of the different components produce slightly asymmetric changes in the mechanical dimensions. We minimize these effects by trimming the amplifier gain so that its temperature coefficient balances the temperature sensitivity of the sensor. The overall temperature sensitivity is balanced to zero in first order, but the system has a residual sensitivity to temperature gradients. We minimize the size of these gradients by mounting the instruments at the bottoms of boreholes which provide very stable temperature environments.

The boreholes are typically 30 m deep, although we have experimented with different depths ranging from 15 m to 122 m. They are nominally 15 cm in diameter and are cased with steel casing over their entire length. In addition to providing an isothermal environment, the installation at depth attenuates the large rainfall-induced tilts of the near-surface material. Even moderate rains can cause unconsolidated material within a few meters of the surface to tilt at a rate approaching 2 μrad/day.

The magnitude of the power spectrum measured by the instrument[9] is about 8 X 10^{-15} $(rad)^2/Hz$ at semi-diurnal periods and decreases approximately as the square of the period for shorter periods down to about 2 s. The pendulum has a mechanical resonance at about 0.7 s, and its calibration becomes somewhat uncertain as the resonance is approached. Note that the tilt and strain noise power spectra are comparable in magnitude when the fluctuations in both are expressed in the same form of a dimensionless quantity per unit bandwidth. This rough equality may not be preserved at higher frequencies as horizontal accelerations become a more important component of the tilt signal.

5. VERTICAL ACCELERATION

We have also conducted studies of the spectrum of vertical vibrations. These observations[10] use gravity meters, which respond to the vertical component of the effective acceleration of gravity. Many different types of instruments are available, but most relative gravity meters monitor the position of a mass that is suspended on some kind of spring. The position of the mass is sensed using something like the capacitance bridge arrangement described above for a tiltmeter.

Gravity meters are sensitive in first order to barometric pressure changes (because of the direct attraction of the air mass overhead), and they are also sensitive to changes in the sub-surface water level for the same reason. The mechanical components have the same kinds of problems that we described above for tiltmeters. The long-term performance of a relative instrument is likely to be limited by the stability of the mechanical support. All of these effects begin to become significant problems when measuring vibrations with periods longer than a few minutes.

The spectrum of vertical vibrations tends to show significant variability from site to site; when expressed in dimensionless units of $\Delta g/g$, the low-frequency power spectrum is likely to be on the same order as the tilt spectrum quoted above, but local effects are often quite large and global averages may not be very useful in predicting the noise at a particular site.

6. INTERPRETATION OF THE MEASUREMENTS

Tilts, strains and vertical displacements are physically different effects, and they are observed with different instruments as we have discussed. They often become coupled in the real earth, however, especially in a region that is inhomogeneous or anisotropic[11]. A discontinuity in the elastic parameters of the earth usually results in a strain discontinuity as well since the stress must be continuous across the boundary if the material is to remain in mechanical equilibrium. Two points on opposite sides of the boundary are therefore displaced by different amounts, and a line joining those two points will be both translated (resulting in an anomalous strain) and rotated (resulting in a strain-induced tilt) in the general case. The resulting anomalous strains and tilts may be very large if the elastic transition is abrupt near the boundary. The strain may also cause material in the softer region to be uplifted, but the resulting change in the local acceleration of gravity is usually quite small since the uplift must be scaled by a length comparable to the radius of the earth (rather than by the width of the transition zone which is the important scale length for strain and tilt).

The coupling coefficients between strain and tilt depend on the details of the elastic discontinuity but generally not on the frequency of the driving stresses. Quantitative measurements of tilt spectra must therefore be corrected for tilt-strain coupling using coefficients that are usually derived from finite-element models of the region of interest. The coupling coefficients may have substantial uncertainties because the exact shapes of the elastic discontinuities are often not well known. The coupling coefficients can be quite large near a boundary and can approach 50% at sites which have a surface weathering layer that overlies a more competent deeper layer with significantly different elastic parameters.

Measurements of strain can be affected by the presence of cracks or fractures in the medium. The fracture is so much weaker than the surrounding material that almost all of the strain displacement appears across it. The strain in the vicinity of the crack varies rapidly with the distance from it under these circumstances; the details of this variation can be estimated using from a model constructed using finite-element techniques. As with tilt, these effects are generally independent of the driving frequency over a wide range of excitations and are therefore *not* limited only to the long-period effects that are usually studied in the geophysical context. The rapid spatial variation produced by cracks may be attenuated to some degree by building a long-baseline instrument that responds to the spatial average of the strain component.

These effects make it difficult to characterize the vibration level in a regional sense -- there may be many very short range effects that magnify the local strain, result in strain-induced tilts and introduce a large spatial variation into what we might naively have expected to have been a smoothly-varying function.

7. CONCLUSIONS

Measurements of low-frequency vibrations of the earth are simple in principle, but most of the instruments with enough sensitivity to observe these motions are also sensitive to a number of spurious effects including fluctuations in ambient temperature and barometric pressure. It is quite difficult to completely eliminate these problems, and the difficulties tend to increase with the period of the signal being observed. We have described a number of solutions to these measurement problems.

A second difficulty is the ambiguity of displacement measurements made in the inhomogeneous and anisotropic real earth. This complex environment tends to couple tilt and strain (and gravity, to a lesser extent); tilt data are substantially affected by the interaction between the strain field and the inhomogeneities of the region; strain measurements show large and rapidly varying spatial variations which complicate the interpretation of measurements made at only a few points in a region. Measurements of the acceleration of gravity are less affected by these problems, but they do not provide as much information as either strain or tilt, so that they may be less useful in predicting the actual noise spectrum at a given point. These effects are independent of the frequency of the motion being observed over a wide range of frequencies and are not limited to the long-period measurements which are commonly made in geophysically-motivated studies of the earth.

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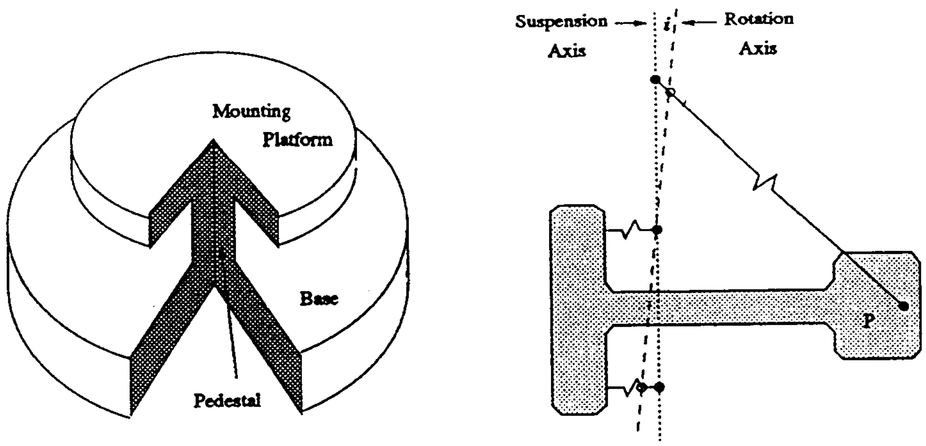

(a) (b)

Fig. 1. a) Schematic drawing of the mounting plate composed of the based and the mounting platform, and b) schematic of the horizontal pendulum, P. The angle i between the rotation and suspension axes is 2.3°. The pendulum is supported using three springy wires, shown as solid lines.