LIGHT SCATTERED FROM TWO ATOMS*

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Abstract

We have observed interference fringes, like those in Young's classic experiment, in the laser light scattered by two trapped atoms. The interference fringes are present only in one polarization of the scattered light. The polarization dependence is related to the complementarity principle, which forbids the simultaneous observation of wave-like and particle-like aspects of light. The interference fringes are due to processes in which a single photon scatters from two atoms. We describe methods which might be used to observe other interference effects due to two photons scattering from two atoms.

Introduction

We have observed Young's interference fringes resulting from the light scattered from two atoms. In Young's original experiments, sunlight, passing through two slits, produced a pattern of closely-spaced fringes on another screen. Those observations established the wave nature of light and resulted in the first accurate determination of the wavelength of light.[1]

Interference experiments of this form have played an important part in the conceptual development of quantum mechanics. Either matter (electrons, for example) or light can display interference (a wave-like property). However, if it is possible to infer which slit the matter or light passed through, thereby extracting a particle-like property, interference is not observed. Wave-particle duality is a particular example of a more general principle, the principle of complementarity, which states, "For each degree of freedom, the dynamical variables are a pair of complementary variables."[2] Variables are complementary if a precise determination of one implies an inability to predict the other. For example, the position and momentum of a particle are complementary variables, since, according to Heisenberg's uncertainty principle, the product of their uncertainties must be greater than $\hbar/2$. In our experiments, in contrast to various *gedanken* experiments, complementarity is enforced in a manner which does not require the position-momentum uncertainty relations.

Experimental Apparatus

Figure 1 shows the experimental apparatus, which has been described previously.[3] A linear rf trap[4] was used to confine $^{198}Hg^{+}$ ions. In $^{198}Hg^{+}$, the lowest-frequency electric-dipole transition is from the ground $6s\,^2S_{1/2}$ level to the $6p\,^2P_{1/2}$ level at 194 nm. The ions were laser-cooled to temperatures of a few millikelvins with a beam of linearly polarized, continuous-wave light. Cooling in the trap resulted in strong localization of the ions, which is essential for observation of interference fringes. The trap potentials were arranged so that a pair of ions would be oriented along the symmetry (z) axis of the trap. The laser beam intersected the ions at an angle θ of 62° with respect to the trap axis. Light emitted by the ions was focused by a lens L through an aperture A and then directed to the surface of an imaging detector D_2. This detector was used

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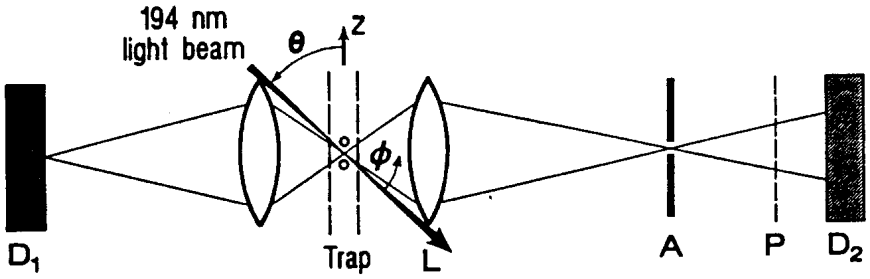

Figure 1: Diagram of the experimental apparatus.

to observe the fringes. The axis of the detection optics was in the plane defined by the laser beam and the trap axis. The in-plane detection angle ϕ varied from 15° to 45° with respect to the laser beam direction. The out-of-plane detection angle varied from -15° to +15°. Optionally, a linear polarizer P was inserted before D_2. Another lens system formed a real image of the ions on another imaging detector D_1. This image was used to determine when there were precisely two ions in the trap.

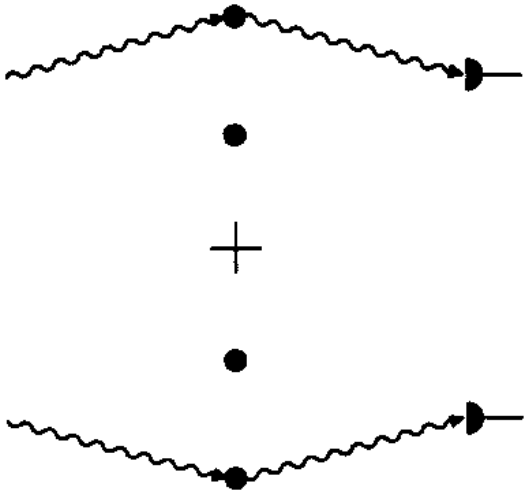

Figure 2: The two paths which contribute to the Young's interference fringes. The paths correspond to the photon being scattered by one atom or the other and then being detected.

One-Photon Two-Atom Scattering

Our version of Young's interference experiment, in which the two slits are replaced by two atoms, is an example of a case in which each photon interferes only with itself. (We call this second-order interference, i.e., second-order in the fields, in order to distinguish it from other forms of interference to be discussed later.) From the point of view of quantum mechanics, we expect interference whenever there are two or more possible paths from a given initial state to the same final state. The absolute square of the sum of the complex amplitudes assigned to these paths yields the probability for the final state. The two paths which yield the interference are represented by the two parts of Fig. 2. In each path, the photon, represented by the wavy arrow, is scattered by one of the two atoms, represented by circles, and reaches the detector,

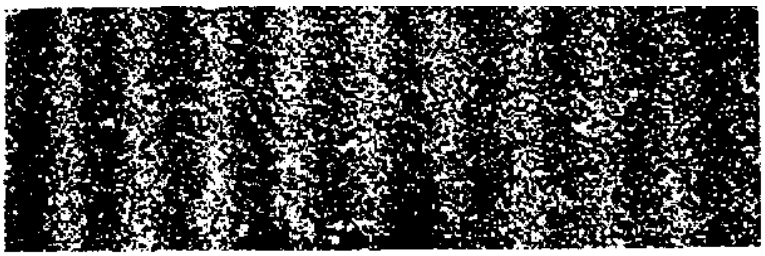

Figure 3: Interference fringes observed for an ion separation of 3.4 μm. The deviation from the forward scattering direction increases to the right.

represented by the half circle. After the photon is detected, there is no way to tell which atom scattered it, i.e., which of the two paths the system took.

An example of interference data is shown in Fig. 3. The two ions were separated by 3.4 μm. The angular separation of successive fringes is about 3°. The angle ϕ between the incident and scattered photon directions increases to the right. Interference fringes were obtained for ion separations varying from 9 μm to 3.3 μm. The angular separation of the fringes increased as the distance between the ions was decreased. The fringe visibility was observed to decrease with increasing ϕ.

The dependence of the fringe spacing on the ion separation and the variation in the fringe visibility with angle are contained in the following expression for the intensity I of the scattered light as a function of $\vec{q}$, where $\hbar\vec{q} = \hbar(\vec{k}_{\rm out} - \vec{k}_{\rm in})$, and $\vec{k}_{\rm in}$ and $\vec{k}_{\rm out}$ are the wavevectors of the incoming and outgoing photons:

$$I(\vec{q}) = 2I_0 \left[1 + \cos(q_z d) \exp\left\{-\left\langle[\vec{q}\cdot(\vec{u}_1 - \vec{u}_2)]^2\right\rangle/2\right\}\right]. \quad (1)$$

In this expression, I_0 is the intensity due to scattering by a single ion, d is the ion separation, and $\vec{u}_1$ and $\vec{u}_2$ are the displacements of the ions 1 and 2 from their equilibrium positions. The angular brackets denote an ensemble average. Equation (1) was derived by methods similar to those used for Bragg scattering by a harmonic crystal.[3, 5] The fringe separation is contained in the factor $\cos(q_z d)$ and is therefore inversely proportional to d. The fringe visibility depends on the quantity $\left\langle[\vec{q}\cdot(\vec{u}_1 - \vec{u}_2)]^2\right\rangle$. The visibility is greatest in the forward scattering direction, where $\vec{q} = 0$, and decreases with increasing scattering angle and with increasing ion temperature. The temperature of the ions has been determined from the rate of decrease in fringe visibility with angle and is approximately equal to the theoretically calculated value.[3]

The data shown in Fig. 3 were obtained with polarization-insensitive detection. When a linear polarizer was placed in front of the detector, the fringes were present only in the light having polarization in the plane defined by the polarization of the incident light and $\vec{k}_{\rm out}$ (π-polarization) and not in the light having the orthogonal polarization (σ-polarization). Figure 4 shows interference patterns observed in (a) π-polarized and (b) σ-polarized light. The π-polarization shows high fringe visibility, while the σ-polarization shows no fringes, only the slow variation with scattering angle of the detection sensitivity.

The source of the polarization dependence of the fringe visibility lies in the internal level structure of the $^{198}Hg^+$ ion and in basic principles of quantum mechanics. Figure 5 shows the magnetic sublevels involved in the $6s\,^2S_{1/2}$-to-$6p\,^2P_{1/2}$ transition. The static magnetic field is small enough that we are free to define the quantization axis of the ions to be along the electric polarization vector of the incident light. If the static magnetic field is along some other direction, then the Zeeman sublevels defined according to the electric polarization vector are not stationary states. This does not change the analysis as long as the Zeeman precession frequency is much less than the inverse of the scattering time, which is approximately equal to the $6p\,^2P_{1/2}$-state lifetime (2.3 ns). In the experiments described here, the magnetic field was small enough that this was always the case.

The π-transitions ($\Delta m_J = 0$) and σ-transitions ($\Delta m_J = \pm 1$) are labelled in Fig. 5. The incident laser light drives only π-transitions from the ground to the excited state. However, the decay from the excited

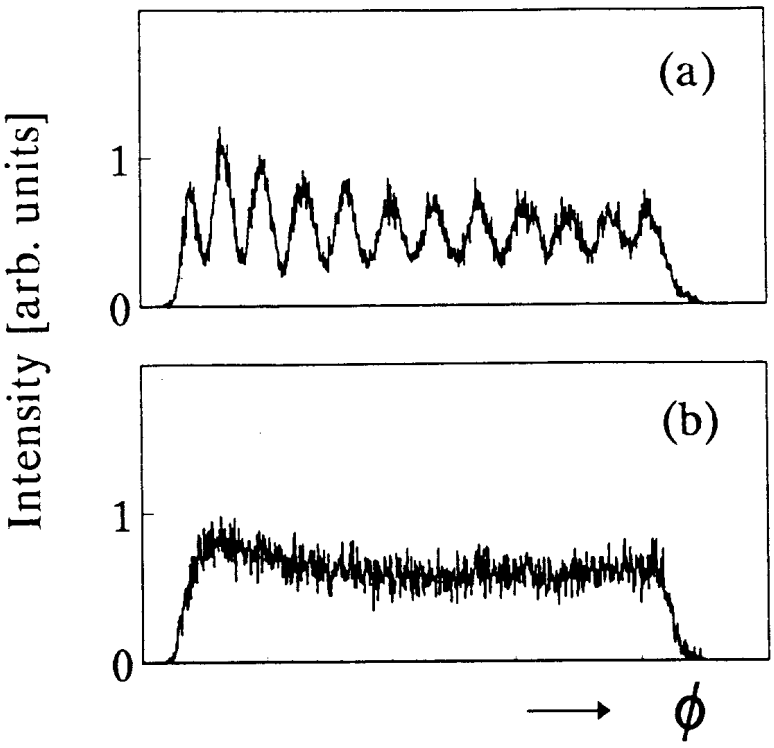

Figure 4: Intensity of the scattered light from two ions as a function of the scattering angle ϕ. (a) π-polarized light. (b) σ-polarized light. Only the π-polarized light shows interference fringes. The data are not normalized for the detection efficiency, which varies with ϕ.

state to the ground state can be either a π-transition or a σ-transition. If the decay is a π-transition, the atom returns to the same m_J sublevel it occupied before it absorbed a photon. Hence, it is not possible to tell, by examining the atoms, which one scattered the photon. On the other hand, if the decay is a σ-transition, the atom returns to a different m_J sublevel and it would be possible in principle to examine the atoms after the scattering and tell which one scattered the photon. In the context of the principle of complementarity, a particle-like property is present, and interference, a wave-like property, must vanish.

From a more modern point of view, we say that interference occurs when there is more than one transition amplitude connecting the initial and final states.[6] Figure 6 illustrates this point of view. There are four possible initial states of the combined system of two atoms, since each atom can be in either of two m_J sublevels. We consider a particular initial state, in which both atoms are in the $m_J = +1/2$ sublevel. All four choices of initial states contribute to the interference fringes in the same way. In Fig. 6(a), a π-polarized photon is absorbed and a π-polarized photon is emitted. There are two paths for this process. Since they lead to the same final state, interference can occur. In Fig. 6(b), a π-polarized photon is absorbed and a σ-polarized photon is emitted. There are two paths for this process. However, they lead to different final states, so there is no interference.

Polder and Schuurmans[7] calculated the spectrum of resonance fluorescence from a $(J = 1/2)$-to-$(J = 1/2)$ transition. They found that, for low intensities, the π-polarized scattered light is coherent with the incident light, while the σ-polarized scattered light is not. Thus, it is reasonable that the π-polarized light should show interference fringes while the σ-polarized light should not. The fringe visibility for the π-polarized light should decrease with intensity, since, as is the case for two-level atoms,[8, 9] the ratio of incoherent scattering

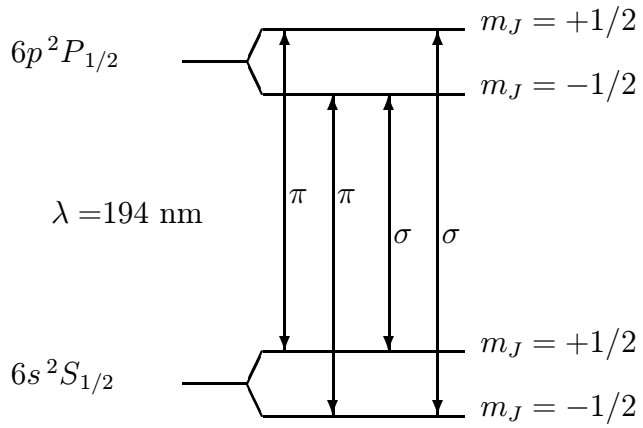

Figure 5: Zeeman sublevels involved in the 194 nm, $6s\,^2S_{1/2}$-to-$6p\,^2P_{1/2}$ transition of $^{198}Hg^+$. The allowed π and σ transitions are labeled. The Zeeman splitting of the levels is exaggerated.

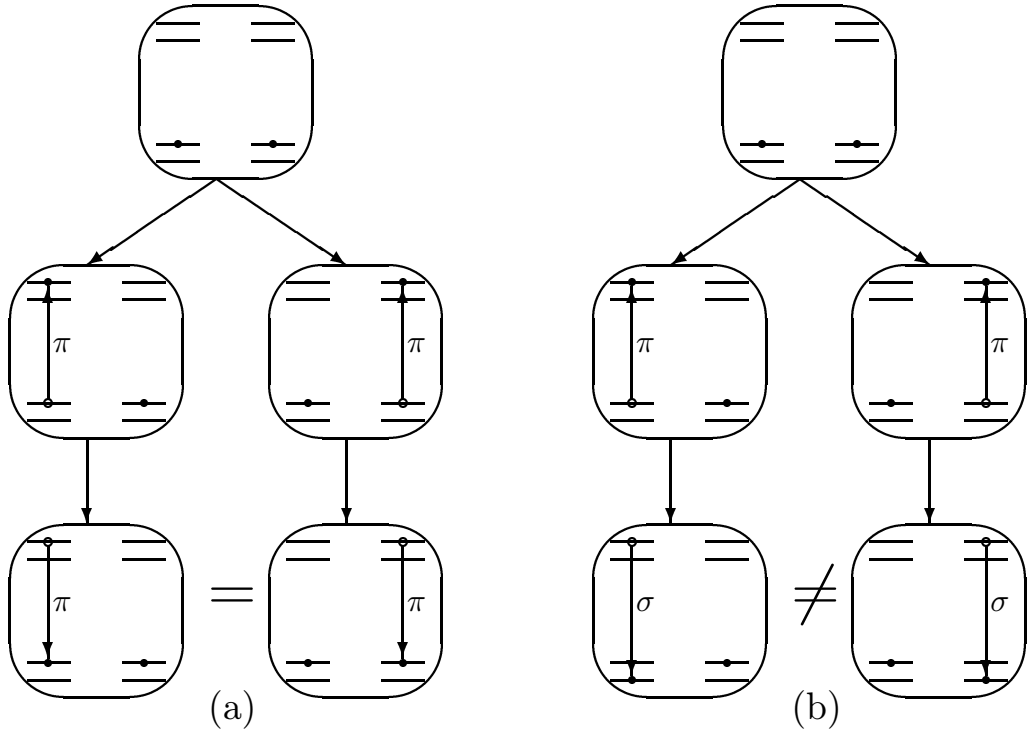

Figure 6: Each box represents the combined state of the two atoms. The ordering of energy levels is the same as in Fig. 5. In (a), a π-transition is made from the ground state to the excited state and is followed by a π-transition back to the ground state. There are two possible paths, both of which lead to the same final state, so that interference is possible. In (b), the π-transition from the ground to excited state is followed by a σ-transition back to the ground state. The two paths do not lead to the same final state, so there is no interference.

to coherent scattering increases with intensity. The intensity dependence of the visibility has not yet been examined experimentally.

Two-Photon Two-Atom Scattering

Next, we consider the interference process represented by Fig. 7. Two photons are scattered by two atoms and are each detected at different spatial positions. However, it is not possible to tell which atom scatters which photon, so there is interference between the two paths. This is a kind of fourth-order interference (i.e., fourth-order in the fields), which can persist even when there is no second-order interference. The interference effects that we have labeled fourth-order and second-order are called second-order and first-order, respectively, by others,[9] since they are second-order and first-order in the intensities.

Interference by independent quantum sources has been discussed in detail by various authors.[10, 11] If the sources are independent, there is no stationary interference pattern. However, both classical and quantum calculations predict intensity correlations between two detectors. In particular, if the sources are two single atoms, there will be a finite coincidence rate if the detectors are separated by n interference fringes (n=0, 1, 2, ...) and no coincidences if the detectors are separated by $n + 1/2$ fringes.[10] In this case, the quantum calculation predicts a fringe visibility of 1, while the corresponding classical calculation predicts a visibility less than or equal to 1/2. The difference arises from the fact that a single atom cannot emit two photons simultaneously. After emitting one photon, it must be excited to the upper state again before it can emit another. In the classical calculation, the simultaneous detection of two photons emitted by the same atom is allowed, and this gives rise to a background signal that reduces the fringe visibility. The observation of a fringe visibility greater than 1/2 for independently phased atoms would thus be an example of a quantum phenomenon having no classical analog.

Consider a simple case originally treated by Dicke.[12] Two two-level atoms are both in the excited state and are separated by much more than the wavelength of the light that they emit (see Fig. 8). This state can be written as $|\mathrm{e}\rangle_1|\mathrm{e}\rangle_2$, where e denotes the excited state of an atom and 1 and 2 label the two atoms. The first photon can be emitted in any direction consistent with the dipole radiation pattern of a single atom. The state of the atoms immediately after the first photon has been emitted can be symmetric, i.e., $(|\mathrm{e}\rangle_1|\mathrm{g}\rangle_2 + |\mathrm{g}\rangle_1|\mathrm{e}\rangle_2)/\sqrt{2}$, or antisymmetric, i.e., $(|\mathrm{e}\rangle_1|\mathrm{g}\rangle_2 - |\mathrm{g}\rangle_1|\mathrm{e}\rangle_2)/\sqrt{2}$, where g denotes the ground state of an atom. In general, the state can be a linear combination of these two cases.

Like the radiation pattern of pair of classical dipoles, the probability distribution for the emission of the second photon is made up of many interference fringes. A pair of classical dipoles has a pattern which shifts according to their relative phases. Here, the pattern shifts according to the relative phases of the wavefunctions of the two atoms. Figure 8 shows an example, for a separation of five wavelengths. If the wavefunction is symmetric with respect to the two atoms, the radiation pattern is given by the solid curve. If it is antisymmetric, it is given by the dashed curve. If it is a linear combination of symmetric and antisymmetric wavefunctions, it is intermediate between the two curves.

The direction of the second photon is constrained by the direction of the first. It cannot be in a direction differing by $n + 1/2$ fringes and has the greatest probability of differing by n fringes. The emission of the first photon fixes the symmetry of the wavefunction and hence the probability distribution for the emission of the second photon. The symmetry must be such that the direction of the first photon is along a fringe (lobe) of the radiation pattern. It is important to remember that each photon is emitted by *both* atoms. If, after the emission of the first photon, a measurement is made to determine which of the atoms is in the excited state, the coherent superposition is totally destroyed, and the second photon can be emitted in any direction.

Consider the following modification of the experiment. Wait until the first photon is detected. Its direction of emission fixes the symmetry of the wavefunction. Then, before the second photon is emitted, change the separation between the atoms to a different value. In an ion trap, this can be done by changing the electric potentials on the electrodes. This changes the radiation pattern, adding or subtracting fringes. The second photon will be emitted into a fringe of the new radiation pattern. Thus, the second photon can be aimed toward certain directions and away from others by controlling the distance between the atoms.

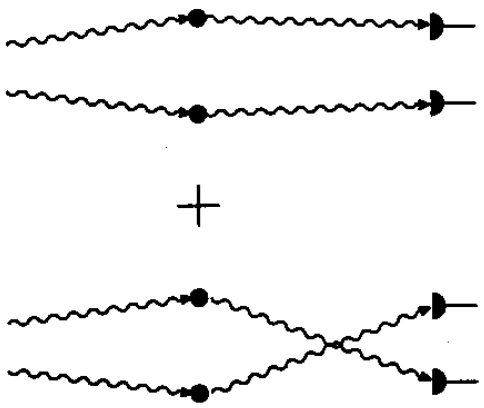

Figure 7: The two paths which contribute to fourth-order interference fringes. The paths correspond to each of the two photons being scattered by one of the two atoms and then being detected by one of the two detectors.

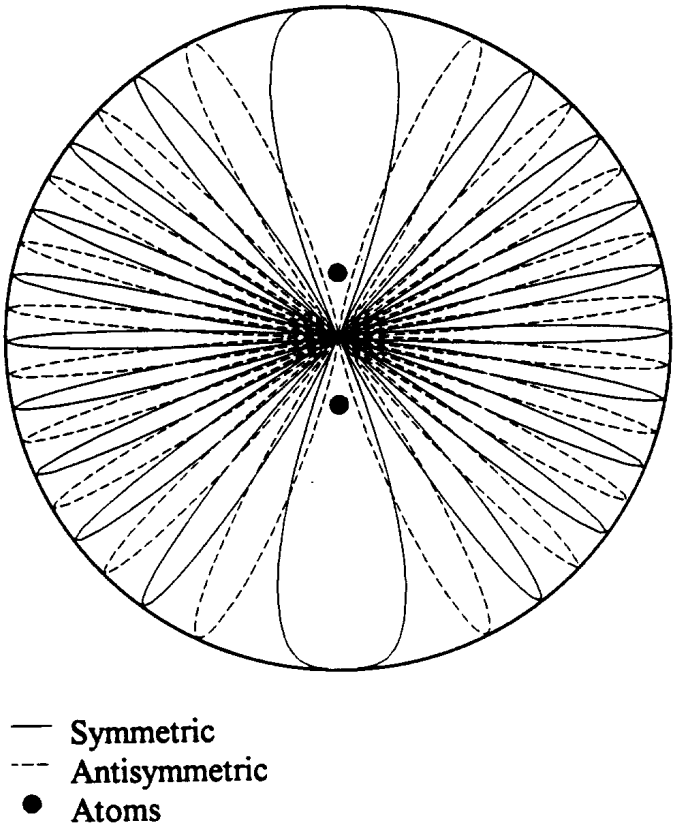

Figure 8: Radiation patterns for two atoms (indicated by circles) separated by 5 wavelengths and initially both in the excited state. The solid curve is for the case in which, after the emission of one photon, the atoms are in a symmetric state. The dashed curve is for the case in which they are in an antisymmetric state.

Estimated Signals

Experimentally, a higher data-collection rate could be achieved by exciting the atoms continuously and using coincidence detection, rather than by sequentially exciting the atoms and then detecting the emitted photons. It should be possible to observe fourth-order interference with a slight modification of the present experimental apparatus. Two imaging detectors would be used, together with fast coincidence electronics. In order to avoid the intensity variations due to the second-order interference [as in Fig. 4(a)], only the σ-polarized light should be detected. For each coincidence, the position of both photons would be recorded.

Let $R_{\rm coinc}$ be the detected coincidence rate. Then,

$$R_{\rm coinc} = 2(\eta R_{\rm scatt})^2 \delta\tau, \tag{2}$$

where η is the fraction of scattered photons that are detected, $R_{\rm scatt}$ is the rate at which photons are scattered by one atom, and $\delta\tau$ is the coincidence time window. A reasonable value for η is 10^{-3}, assuming a detection solid angle of 1% and a detector quantum efficiency, including the polarizer efficiency, of 10%. If the incident light intensity is high enough to saturate the transition, $R_{\rm scatt}$ approaches $\gamma/2$, where γ is the decay rate of the upper state. The coincidence time window $\delta\tau$ should be less than the coherence time for the fourth-order interference, which is less than or approximately equal to γ^{-1}. If we let $\delta\tau \approx 0.25\gamma^{-1}$, then

$$R_{\rm coinc} \approx 1.25 \times 10^{-7}\gamma. \tag{3}$$

For Hg^+, where $\gamma \approx 4.3 \times 10^8$ s^{-1}, $R_{\rm coinc} \approx 54$ s^{-1}. Thus, it should be possible to observe a fringe pattern with a few minutes of observation time.

Other ions might be better suited than Hg^+ for this experiment. For example, Mg^+ has a smaller mass, thus allowing better spatial localization, and a longer resonance wavelength, thus allowing the use of more efficient detectors and polarizers. On the other hand, it has a smaller value of γ than Hg^+ ($\gamma \approx 2.7 \times 10^8$ s^{-1}).

It would also be of interest to observe the coincidences in the π-polarized light, particularly in the high-intensity limit. In this limit, the second-order interference disappears, but the fourth-order interference remains.[9] The visibility of the fourth-order interference fringes is 1 for arbitrary intensity, but the time window during which it can be observed decreases with intensity.

Quantum Erasers

The term "quantum eraser" was used by Scully and Drühl[13] to describe a *gedanken* experiment involving the observation of interference fringes in the light scattered by two atoms. Other forms of quantum erasers have been described.[14, 15] The common element is that interference is destroyed when it is possible to tell, in principle, which path the system has taken, and is recovered when that information is erased or hidden. In Scully and Drühl's original proposal, two atoms are excited by a laser pulse. In the case of two-level atoms, interference fringes are observed, as in the case of π-polarized emission discussed in the previous Section. In the case of three-level atoms, interference is not observed when the light emission from the atoms is accompanied by a transition to a level different than the one occupied before the laser pulse. This is because it is possible to tell which atom scattered the photon, as in the previous case of σ-polarized emission. This information is erased by exciting the atoms to a fourth level, from which they decay back to the initial level. Fringes are recovered, but only in a coincidence measurement, which is in principle similar to the fourth-order interference experiment that we have proposed. The experiment can be operated in a "delayed choice" mode. That is, the interference fringes can be made to appear or not, depending on whether or not a shutter is operated, even though this is done after the photon is on its way to the detector.

The fourth-order interference experiment we have proposed is a kind of quantum eraser, though, in common with some other quantum erasers,[14, 15] it lacks the "delayed-choice" feature. If we detect only the σ-polarized light, no interference fringes are detected in a non-coincidence experiment. In principle the states of the atoms could have been measured before and after the photon scattering, and it would be possible to tell which atom scattered the photon. In a coincidence detection, *both* atoms have scattered a photon and

have changed their states. However, it is impossible to tell *which* atom scattered *which* photon. That is, it is impossible to tell which of the paths shown in Fig. 7 the system took. This is what makes it possible to observe interference fringes.

Conclusion

We have observed Young's interference fringes in the light scattered from two localized atoms. The interference patterns can be used to infer the separation and temperature of the two atoms. The fact that the interference appears only for one polarization of the light has a simple explanation based on the fact that only paths leading to the same final state can interfere. Fourth-order interference effects might be observed by detecting coincidences in two photon detectors.

This work was supported by the Office of Naval Research.

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