INTRODUCTION

In these notes, we discuss recent experiments conducted by the ion storage group of the National Institute of Standards and Technology in Boulder, Colorado. The overall goal of this work has been the development of techniques for high resolution spectroscopy using stored ions. These techniques have also been applied to problems of practical and fundamental interest. In the following we summarize the work for the year preceding July, 1991.

(1) $^{9}$Be$^{+}$ Hyperfine pressure shift

In this experiment, an oscillator has been locked to the $(m_{l} = -1/2, m_{J} = 1/2) \leftrightarrow (-3/2, 1/2)$ nuclear spin-flip, hyperfine, clock transition ($\omega_{\Omega}/2\pi = 303$ MHz) in the ground state of $^{9}$Be$^{+}$ (Fig. 1). The details of this experiment have been described previously [1].

![Fig. 1. Hyperfine energy levels (not drawn to scale) of the $^{9}$Be$^{+}$ 2s $^{2}S_{1/2}$ ground state as a function of magnetic field (horizontal axis). At $B = 0.8194$ T the 303 MHz clock transition is independent of magnetic field to first order.](image)

A shift of the clock transition frequency, with an unexpectedly large value, was discovered when the background gas pressure was increased [1]. The background gas pressure could be increased by moving the magnet of the sputter ion pump (which evacuated the trap region) so that it overlapped fewer pumping cells and reduced the pumping speed. We checked to make sure the magnetic field at the site of the ions was not disturbed.

The composition of the gas was not known since the pressure was measured with a Bayard-
It was difficult to install a residual gas analyzer on the system without a thorough reprocessing of the vacuum system. Therefore we opted to leak in various gases which are known to be present in high vacuum systems [2] that are similar to our \(^{9}\text{Be}^+\) apparatus. We monitored the frequency of the oscillator that was locked to the clock transition as an increase in the background gas pressure was separately provided by the following gases: \(\text{H}_2\), \(\text{N}_2\), \(\text{O}_2\), \(\text{H}_2\text{O}\), \(\text{CO}\), \(\text{He}\), \(\text{CH}_4\), and \(\text{CO}_2\).

The maximum pressure for any gas introduced into the system was around \(4 \times 10^7\ \text{Pa}\) (3 \(\times 10^9\) Torr), and the base pressure of the system (as measured by a Bayard-Alpert gauge calibrated for \(\text{N}_2\)) was \(1.3 \times 10^8\ \text{Pa}\) (1 \(\times 10^{-9}\) Torr). (Below, we use pressure readings of the Bayard-Alpert gauge which was calibrated for \(\text{N}_2\); the quoted results must be corrected for changes in sensitivity for the various gases.) The gases \(\text{H}_2\), \(\text{CO}\), \(\text{H}_2\text{O}\), \(\text{He}\), \(\text{O}_2\), and \(\text{N}_2\), gave no observable pressure shift. Specifically, the gases \(\text{H}_2\), \(\text{CO}\), and \(\text{N}_2\) caused fractional frequency shifts of the \(^{9}\text{Be}^+\) clock transition less than \(1.5 \times 10^4/\text{Pa}\) (2 \(\times 10^{14}/10^{10}\) Torr). Similarly, the gases \(\text{H}_2\text{O}\), \(\text{He}\), and \(\text{O}_2\) caused fractional frequency shifts less than \(3.8 \times 10^7/\text{Pa}\) (5 \(\times 10^{13}/10^{10}\) Torr). The gas \(\text{CH}_4\) gave a pressure shift of \((-1.7 \pm 0.4) \times 10^9/\text{Pa}\) ((-2.2 \(\pm 0.5) \times 10^{13}/10^{10}\) Torr). When \(\text{CO}_2\) was admitted to the system the hyperfine frequency slowly changed (over about one hour) as though \(\text{CO}_2\) slowly reacted with some constituent inside the vacuum system and released a gas which caused a pressure shift of the clock transition.

The large difference between our data for \(\text{CH}_4\) and the other gases we measured, and the difference between our data for \(\text{CH}_4\) and ion hyperfine pressure shifts induced by inert gases which were measured by other groups [3,4], is not understood at this time. One possible explanation is suggested by studies [5] of radiative association of \(\text{C}^+\) with \(\text{H}_2\) to form \(\text{CH}_2^+\). In the models of this process, it is assumed that the \(\text{H}_2\) can stick to the \(\text{C}^+\) for a long enough time to allow the \(\text{C}^+\text{-H}_2\) complex to radiatively stabilize. This sticking is possible because the collision energy can be taken up by the internal degrees of freedom in the \(\text{H}_2\) molecule or the \(\text{H}_2\text{-C}^+\) complex. The sticking time can be orders of magnitude longer than the interaction time during a simple elastic collision. If a similar sticking mechanism is active in \(\text{CH}_4\text{+ Be}^+\) collisions, it may account for the apparent large pressure shift.

This pressure shift might affect all precision measurements of hyperfine structures of ions similarly. That is, we might expect the sticking time to depend only on the charge of the ion. In that case, it may be necessary to achieve significantly better vacuums by using cryopumping.

(2) Linear Paul Trap

The main advantage of using a single ion in a Paul quadrupole trap is that the kinetic energy of micromotion can be on the order of the secular motion energy. For a single \(^{199}\text{Hg}^+\) ion cooled to the Doppler-cooling limit, the second-order Doppler shift would be \[<\Delta \nu_{\text{Doppler}}; \nu_s> = -2.3 \times 10^{-18}.\] In a quadrupole ion trap, two or more ions in the trap are pushed from the center of the trap by their mutual Coulomb repulsion. Therefore, the second-order Doppler shift is higher due to increased micromotion.

Consider the trap shown in Fig. 2. In this trap, the rf electric fields are transverse to the trap axis for the entire \(z\) extent of the trap. If a single string of ions is trapped along the \(z\) axis, the kinetic energy of micromotion is about equal to the kinetic energy in the secular motion. This kind of trap was first demonstrated for atomic ions by Drees and Paul [8]; lower temperatures were later obtained by Church [9]. Prestage et al. [10] have demonstrated a \(^{199}\text{Hg}^+\) microwave clock with excellent short term stability \((\sigma_t(t) = 1.6 \times 10^{13} \tau^{10})\) using a cloud of ions elongated in the \(z\) direction in a linear Paul trap. Dehmelt first suggested using a string of ions to suppress the second-order Doppler shift in such a trap [11]. Strings of atomic ions have been observed at Garching [12] and NIST [13].

In Fig. 3a, we show a string of \(^{199}\text{Hg}^+\) ions confined in a linear trap that has rod diameters...
1.60 mm and distance of the rod centers from the $z$ axis of the trap equal to 1.55 mm. The spacing of the ions is approximately 20 $\mu$m.

Fig. 2. Linear trap configuration. The alternating rf voltage $V_0 \cos \Omega t$ is applied to diagonally opposing electrodes as shown. We assume the end portions of the electrodes are long enough that the resulting rf potential at the position of the ions is independent of $z$, so that the rf electric fields are parallel to the $x$-$y$ plane. To trap ions along $z$, the center four electrodes are held at static ground potential and the two sets of four electrodes on either end are held at a static potential $U_o$ ($U_o > 0$ to trap positive ions). The average position of the ions could be made to coincide with the rf electric field null by applying slightly different static potentials to the four central rods to correct for offsets from contact potentials or static charge on the electrode surfaces. This geometry allows laser beams to be directed along the $z$ axis.

When the linear density of ions is increased by increasing the static voltage applied to the end sections of the rods, or when the $x$-$y$ potential is weakened, zig-zag structures like those shown in Fig. 3b result. These structures are the lowest-energy configurations expected for a certain range of the ratio of $z$ confining potential and $x$-$y$ potential. They have been predicted to be the lowest energy configurations in ion storage rings [14].

Strings of ions should be useful for high resolution spectroscopy. If we use imaging techniques, each ion can be treated as an independent atomic clock where the clock transition could be detected with 100% efficiency (using Dehmelt's "electron shelving" scheme [6,15]). If we use 50 ions, a resonance frequency $\omega_J/2\pi = 40.5$ GHz (the ground state hyperfine transition for $^{199}$Hg$^+$), and if we use the Ramsey method of interrogation where the free precession time is 100 s, the frequency stability of this clock "ensemble" should be $\delta\omega_T = 5.5 \times 10^{14} \tau^{-1}$. One or more ions whose positions are localized to less than $\lambda/2\pi$ are also interesting for experiments investigating interference, superradiance and subradiance, and cavity-QED.

Fig. 3. Images taken of crystallized structures of $^{199}$Hg$^+$ ions in a linear rf trap like the one of Fig. 2. In (a), 10 ions form into a string with length 220 $\mu$m. In (b), the the lowest energy configuration for 13 ions is a zig-zag structure. In (a) and (b), the trap conditions are different but the magnification is the same. (Ref. 13.)
The 40.5 GHz hyperfine resonance has been observed on a string of ions and also on a single ion using electron shelving. Currently, we are investigating the possibility of using cryogenic pumping to minimize ion loss due to Hg+* + Hg → dimer formation and to minimize possible pressure shifts in the hyperfine resonance.

3) Penning trap density limitations

We have discovered and investigated a particular limitation to the number and density of charged particles that can be stored in a Penning trap. The energy and angular momentum of a cloud of ^9Be+ ions were controlled with radiation pressure from two different lasers. One of the laser beams was directed through the radial edge of the ion cloud and supplied a radiation pressure torque which increased the ^9Be+ rotation frequency and density. We discovered that a slight misalignment of the trap symmetry axis with the magnetic field would heat the plasma when we attempted to increase the rotation frequency (or density) beyond a certain value [16]. With about 2000 ^9Be+ ions in the trap, we could use this heating to align the trap symmetry axis with the magnetic field to better than 0.01°. Larger ion clouds were even more sensitive to this misalignment. In fact, in this trap, all other known methods of alignment were less sensitive.

By measuring the rotation frequency where the heating occurred as a function of the trap voltage, we were able to associate this heating with the excitation of a collective mode of the plasma by the static field asymmetry produced by a misalignment of the trap electric field axis from the direction of the magnetic field [16]. The mode is called an (ℓ,m) = (2,1) mode. In a (2,1) mode the plasma tilts with respect to the magnetic field and precesses about the magnetic field axis. In a frame of reference rotating with the plasma, there is a particular (2,1) mode which precesses in a direction opposite to the rotation. For a particular value of the rotation frequency, this becomes a static mode (no time dependence in the lab frame) that can be excited by static asymmetries. It is this static (2,1) mode which we observed to limit the plasma density. While this is the lowest-order and therefore the strongest static mode that can be excited, other weaker heating resonances which tended to limit the plasma density to even lower values were observed. These weaker heating resonances are also due to the excitation of coherent modes by static field asymmetries. These static modes have important implications for experiments where storage of large number of charged particles is important. For example, they may provide a limit to the number of antiprotons or positrons which can be stored in a trap. The plasma modes also play an important role in the asymmetry-induced transport which occurs in a Penning trap even at low densities. An understanding of the modes may therefore improve the long term confinement of many ions in a Penning trap.

4) Search for anomalous spin-dependent forces using ^9Be+ hyperfine spectroscopy

The existence of weakly interacting bosons (such as axions) has been suggested previously. Laboratory experiments might detect scalar or pseudoscalar couplings of such particles to matter in the form of new spin-dependent forces. We have used spectroscopy of ^9Be+ to search for anomalous potentials having a dipole-monopole or dipole-dipole character [17]. The first is expected to include terms like

\[ V_{AB}^{D} = \gamma^2 D \mathbf{S}_A \cdot \mathbf{r} \left( 1/r^2 + 1/r^2 \right) \exp(-r/\lambda_{\mathbf{r}}), \]  

where the spin \( \mathbf{S}_A \) (in units of \( \hbar \)) of particle A couples to particle B, \( r \) is the distance between particles, \( \lambda_{\mathbf{r}} \) is the range of the force, and \( D \) is a coupling constant with units of (mass)^{-1}. A dipole-
dipole interaction would be expected to include terms like

\[ V_{AB}^T = \left( \frac{\hbar^2}{c} \right) T \exp(-r/\lambda_\phi) \left[ (\frac{1}{\lambda_\phi r^2} + \frac{1}{r^3}) \overrightarrow{S}_A \cdot \overrightarrow{S}_B \right. \]

\[ \left. - (\frac{1}{\lambda_\phi^2 r} + \frac{3}{\lambda_\phi^2 r^2} + \frac{3}{r^3}) \overrightarrow{S}_A \cdot \overrightarrow{r} \overrightarrow{S}_B \cdot \overrightarrow{r} \right], \]

where the spin of particle A interacts with that of particle B. T has units of \((\text{mass})^{-2}\) and characterizes the strength of the interaction.

We have been able to place experimental limits on D and T by examining the \(^9\text{Be}^+\) hyperfine "clock" transition frequency \(\nu_0\) under various conditions. Particles A are assumed to be either the \(^9\text{Be}\) nucleus or the unpaired outer electron in \(^9\text{Be}^+\). In the search for \(V_{AB}^0\), particles B were taken to be the nucleons in the earth. We looked for a change in \(\nu_0\) between the cases where B was parallel or antiparallel to the vertical direction in the lab. In the search for \(V_{AB}^\perp\), particles B were taken to be the electron spins in the iron pole faces of an electromagnet. We compared \(\nu_0\) when \(B_\phi\) was created by this electromagnet with \(\nu_0\) when \(B_\phi\) was created by a superconducting solenoid (\(S_\phi\) spins absent).

From these data, upper limits on D and T for the electron and neutron have been established and compared to the results of others [17].

(5) Theory of Sisyphus cooling for a bound atom (ion)

Cooling that results from optical dipole forces has been considered for a bound atom (or trapped ion) [18]. Through optical pumping, the atom can be made to feel decelerating optical dipole forces more strongly than the accelerating optical dipole forces. This effect, which has previously been realized for free atoms, is called Sisyphus cooling [19]. A simple model for a bound atom is examined in order to reveal the basic aspects of cooling and heating when the atom is confined in the Lamb-Dicke regime. Results of semiclassical and quantum treatments show that the minimum energy achieved is near the zero-point energy and can be much lower than the Doppler cooling limit. Sisyphus cooling of trapped \(\text{Mg}^+\) and \(\text{Hg}^+\) have been examined theoretically.

(6) Observation of "atomic projection" noise

In spectroscopy, "technical noise," such as laser amplitude fluctuations caused by an unstable power supply, often dominates the noise. These sources of noise can be eliminated by careful engineering. Two examples of more fundamental noise sources are: (1) the detection shot noise in a laser absorption spectroscopy experiment and (2) the fluctuations in signal caused by the fluctuations in the number of atoms in an atomic beam experiment. These sources of noise can also be eliminated or significantly reduced. For example, laser shot noise can, in principle, be reduced by use of squeezed light [20].

In a stored-ion experiment, the number of ions can be held constant thereby reducing the atomic number fluctuations to zero. Also, when absorption is detected using electron shelving [6,15], the detection noise approaches zero since 100% detection efficiency is possible. Because of this immunity from some sources of noise, we have been able to observe, in a clear way, what might be called "quantum projection noise." Basically, this source of noise is caused by the statistical fluctuations in the number of atomic absorbers (ions in our case) which are observed to make the transition in an absorption spectroscopy experiment [21].

For simplicity, assume that the number \(N\) of atoms is fixed and the efficiency of detection of absorption by each atom is 100%. We also assume the resonance in question is driven using the Ramsey method in the time domain where the pulse times are much less than the free precession time.

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For optimum power, where the transition probability on resonance is unity, the probability of driving the absorber from one state to the other is given by [1, 21]

\[ p = (1 + \cos(\omega - \omega_0) T)/2, \tag{3} \]

where \( \omega_0 \) is the resonance frequency, and \( \omega \) is the applied frequency. Upon detection, the average number of atoms that are observed to make the transition is \( pN \) and the rms fluctuations in the number of ions that are observed to make the transition is given by the dispersion of the binomial distribution

\[ \Delta N = \sqrt{Np(1-p)}. \tag{4} \]

We call this the atomic projection noise because it arises from the fluctuations in the state the atom is projected into when a measurement is made. When no excitation occurs (\( p = 0 \)), or when the atom is totally transferred into the other state (\( p = 1 \)), this noise is zero. This is to be expected when the atom is in an eigenstate, since a measurement should always find the atom in that state. The noise is maximum for \( p = \frac{1}{2} \) which, for Ramsey excitation, is at the point of maximum slope of the resonance curve. Interestingly, we can show that, if this is the only source of noise, the sensitivity to frequency fluctuations is independent of where measurements are taken on the Ramsey curve (Eq. (3)). This is because the slope of the resonance curve, \( \partial p/\partial \omega \), which gives frequency sensitivity (the signal), is proportional to \( \Delta N \) as can be verified from Eqs. (3) and (4). Therefore the ratio of signal to noise is independent of \( \omega \). However, if there are any added sources of noise, it is more advantageous to measure at the points of maximum slope (the "half-power" points) for maximum frequency sensitivity.

In Fig. 4, we show the noise observed on a Ramsey resonance taken on the clock transition on approximately 20 \(^9\text{Be}^+\) ions. The method for measuring the curve is described in Ref. 1. The qualitative evidence for the projection noise is the fact that the observed noise is larger on the sides of the Ramsey resonance than on the peaks and valleys of the curve.

![Fig. 4. A Ramsey resonance taken on the "clock" resonance of \(^9\text{Be}^+\) ions in a Penning trap (refer to Fig. 1). The number of ions (approximately 20) was held fixed. At each rf frequency (horizontal scale), 30 measurements were made. The vertical scale is the number of detected photoelectron counts from the ions' fluorescence observed in a 1 second integration time [1]. The dots indicate the means of the measurements. The error bars indicate the standard deviations. We determined, by independent measurements, that the frequency fluctuations of the oscillator driving the clock transition caused a negligible contribution to the observed signal fluctuations. The fact that the standard deviations are higher on the sides of the resonance than at the peaks and valleys is a result of "atomic projection noise."](image)
(7) Precision measurement of the g, factor of Mg+

Penning traps are well suited to precision measurements of g-factors of ions. The required magnetic field is already present. When the magnetic field is generated by a superconducting solenoid designed to have high homogeneity and stability, there is a potential for very high accuracy. Systematic errors can be well controlled because the ions occupy a small volume, over which the field can be very uniform. Further, the ion sample can be moved by adjusting the trap’s electric potentials in order to map out the magnetic field variations.

Previously, we measured the g, factor of the ⁹Be⁺ ion [22]. This was done in a Penning trap that used a nonsuperconducting electromagnet. The frequencies of a magnetic-field-dependent hyperfine transition were measured at the same time as the ion cyclotron resonance and other motional resonances of the ions. From these measurements, and using the ratio of the ion mass to the electron mass, which was known from other experiments, we obtained g, (⁹Be⁺) to an accuracy of about 1.6 parts in 10⁷. This measurement could be improved by the use of a superconducting magnet.

This year, we measured the ratio of the g, factor of the ²⁶Mg⁺ ion to that of ⁹Be⁺. The two types of ions were trapped and cooled simultaneously in nearly the same volume. (The higher-mass ²⁶Mg⁺ ions form a ring around the ⁹Be⁺ ions [23].) A magnetic-field-dependent hyperfine transition in ⁹Be⁺ and the electron spin-flip resonance in ²⁶Mg⁺ were measured, one after the other, by microwave-optical double resonance. From these measurements, we obtained the ratio g, (²⁶Mg⁺)/g, (⁹Be⁺) to an accuracy of 1.3 parts in 10⁸. Combined with our previous measurement of g, (⁹Be⁺), we obtain g, (²⁶Mg⁺) to an accuracy of about 1.6 parts in 10⁷. This agrees with a Hartree-Fock calculation [24] to about 3 parts in 10⁶. The deviation of g, (²⁶Mg⁺) from the g factor of the free electron, which is the physically interesting quantity, is about 3 parts in 10⁸. Our measurements already determine this deviation to about 0.5 %.

The g, factors of the neutral alkali atoms (Li, Na,…) are well known. Comparisons with the g, factors of the isoelectronic positive ions (Be⁺, Mg⁺,…), may reveal systematic trends and lead to a better understanding of calculational methods.

(8) Subharmonic excitation of a single electron

A single electron in a magnetic field can have its cyclotron motion excited by an oscillating electric field. Similarly, a single electron in a Penning trap can have its cyclotron, magnetron, or axial motion excited by an oscillating electric field. Usually, the excitation of these motions is made at a frequency which coincides with the resonance frequency ω₀ of one of the motions of the electron, but excitation at a multiple of the motional frequency is also possible. This excitation of a single electron (whose orbit can be well controlled) is interesting because it is one of the simplest systems in which to study parametric and subharmonic excitation and because it should be possible to study these excitations in both the quantum mechanical and classical regimes in the same system.

We have recently studied excitation of the cyclotron and magnetron motions of a single trapped electron [27] by an electric field which we assume to be of the form

\[ \vec{E} = \frac{g}{\hbar} \sum_{n=0}^{\infty} C_n x^n \cos \omega t . \]  

Because the field is spatially nonuniform (the xⁿ factors), excitation at some harmonic of the fundamental motional frequency is possible; that is, where \( \omega = n\omega_0 \). This is called subharmonic excitation. One example of this process is subharmonic excitation of the magnetron motion of an electron. The magnetron orbit is first excited to a certain radius by a potential, oscillating at a frequency near \( \omega_0 \), which is applied to one electrode of a split ring electrode of the Penning trap. Since the damping of the magnetron motion is extremely small, the orbit size is nearly constant after
this first drive is removed. The orbit size is observed as an anharmonic frequency shift of the axial resonance frequency [28]. Application of an electric field of frequency \( \omega = n \omega_0 \) causes an excitation near \( \omega_0 \) through the \( C_{n+1} \) and \( C_{n+1} \) terms in Eq. (5). In our experiment, this excitation was difficult to observe on the cyclotron motion because the stability of the magnet was poor and because the damping time of the cyclotron motion allowed initial excitation only due to thermal noise. Nevertheless, it was possible to observe excitation for \( n = 3 \) on the cyclotron motion (\( \omega_0 = 2.82 \) GHz) and for \( n = 9 \) on the magnetron motion (\( \omega_0 = 47 \) kHz).

One feature of this kind of system may be of practical importance. For example, if the subharmonic excitation can be detected by observing the motion of the electron at its fundamental frequency, the resulting device acts as a frequency divider [25,26]. That is, we inject a certain frequency into the device and measure a submultiple of the input frequency. For the electron, this parametric down-conversion process can, in principle, be extended to very high orders; that is, it should be possible to inject a laser frequency and read out a microwave frequency (the cyclotron frequency).

9 Observation of time varying radiation pressure forces

This work arose out of attempts to observe the effects of optical dipole forces on trapped ions. One manifestation of dipole forces would be Sisyphus cooling (Section (5)). It should also be possible to observe the dipole force on ions without the effects of optical pumping (required for Sisyphus cooling). One way the dipole force could be observed is to modulate it at a frequency near a motional resonance frequency of the ions and observe the excitation as a change in fluorescence [22], or a change in the orbit size observed on an imaging device. Although not immediately apparent, this excitation has many similarities with the nonlinear excitation of a single electron (Section (8)).

One configuration for observing time-varying optical dipole forces is the following: Suppose we have a single ion localized near the center of the trap. We focus a Gaussian laser beam so that its waist coincides with the center of the trap along the direction of the beam, but we position the beam transversely so that the center of the trap coincides with the point of half intensity on the side of the beam. Specifically, assume the origin of coordinates is at the trap center and the \( z \) axis is the trap symmetry axis. Assume the beam is incident along the \( \hat{y} \) direction, and its center is in the \( z = 0 \) plane, but is located at a position \( x = (\ln2/2)w_0 \), where \( w_0 \) is the Gaussian beam waist. The laser frequency \( \omega \) is assumed to be detuned by an amount \( \delta \) from a particular resonance transition frequency \( \omega_0 \) of the ion (\( \delta = \omega - \omega_0 \)). For \( \delta > 0 \) the dipole force [29] is in the minus-\( x \) direction; for \( \delta < 0 \), it is in the plus-\( x \) direction. If we now modulate the intensity of the laser we modulate the force in the \( x \) direction and can observe the corresponding excitation of the ion. A similar excitation occurs when we use two overlapping laser beams whose incident directions (taken to be along \( \hat{k_1} \) and \( \hat{k_2} \)) are, in general, different. Assume the beams are plane waves. If the frequencies of the beams are the same, the intensity as a function of position goes as \( \cos[(\hat{k}_1 - \hat{k}_2) \cdot \tau] \). If we choose \( \hat{k}_1 - \hat{k}_2 \) along \( x \), then the intensity is spatially modulated in the \( x \) direction. If we now let the frequencies of the lasers \( \omega_1 \) and \( \omega_2 \) be different, then the dipole force in the \( x \) direction is modulated in time at the frequency \( \omega_1 - \omega_2 \). When \( \omega_1 - \omega_2 \) is adjusted near the motional resonance frequency of the ion, the orbit size will increase. This case is closely analogous to the excitation of a single electron by two laser beams [26].

For a traveling wave laser beam, the ion also feels the force along the direction of the beam due to the spontaneous scattering force. If the intensity of the laser is modulated, the spontaneous force is modulated and motional excitation will occur.

In order to see the dipole forces clearly, the effects of the spontaneous force (laser cooling and heating) must be suppressed. This requires that the laser intensity and detuning be sufficiently
large. In our preliminary experiments, our laser intensity was small enough that the spontaneous forces dominated. We were able to observe the axial and magnetron excitation of a small number of $^6$Be$^+$ ions in a Penning trap due to the modulated spontaneous force. A steady state excitation was observed because the spontaneous force was balanced by laser cooling.

(10) Hg$^+$ optical spectroscopy

The basic idea for this experiment has been discussed previously [6]. The goal of the experiment is to lock a laser local oscillator to the 282 nm quadrupole transition in Hg$^+$. To obtain the best performance, it will be necessary to observe the linewidth of the quadrupole transition with a resolution near the natural linewidth of 2 Hz. The main problem in our experiment, as well as all other current optical frequency standard experiments, is to have the laser linewidth be narrower than the inverse of the time it takes to lock the laser to the resonance line.

By using a single, laser-cooled $^{199}$Hg$^+$ ion as an absolute frequency spectrum analyzer, we have measured a linewidth below 40 Hz for a ring dye laser (oscillating at $\sim 5.3 \times 10^{14}$ Hz) that was stabilized to an ultra-quiet reference cavity of high finesse [31]. This is the best absolute stability that has been demonstrated for an optical laser (as well as the highest atomic or molecular Q reported to date). If we try to improve the laser performance by using the spectrum of the single ion, we are limited by the low signal-to-noise ratio and by the lifetime of the ion in the trap.

To more easily study the performance of the cavity-stabilized laser, a second cavity was constructed on an independent optical table to which the frequency of a laser could be locked. A measurement of the noise fluctuations in the electronic error signals indicated that the laser frequency could be stabilized to each cavity to less than 30 mHz. In other words, the frequency of the stabilized laser tracked the resonance frequency of the reference cavity to within 30 mHz. Some authors claim this number to be their laser linewidth, but this is not the relevant linewidth for metrology. The dominating contribution to the laser linewidth comes from the length instability of the reference cavities.

The study was carried out by heterodyning the light from the two independently stabilized laser beams and analyzing the fluctuations in the power spectrum of the beat frequency. We have attempted to isolate the cavities thermally, barometrically, and mechanically. We begin with a cavity constructed from a cylindrical spacer made from an ultra-low expansion

![Fig. 5. Spectrum of the beat frequency between the two independent cavity-stabilized lasers discussed in the text. A linear, relative cavity drift is removed by mixing the beat note with the frequency from a synthesizer which is swept in time. The resolution bandwidth is 30 Hz. Total integration time for this data is 70 s. The 14.6 Hz sidebands are due to a floor vibration (independently detected with a seismometer). The apparent bandwidth is about 30 Hz; the bandwidth of the most stable laser of the two is at least $\sqrt{2}$ narrower.](image-url)
material. The cavity is suspended by two thin wires in an aluminum vacuum chamber that is evacuated to about $10^6$ Pa. ($= 10^4$ Torr). The walls of the vacuum vessel are temperature-regulated to better than 10 mK. The vacuum vessel is supported by, but thermally isolated from, a stiff surface plate. We have attempted several passive schemes to isolate the surface plates from seismic noise. The best isolation was achieved by supporting the surface plate from the ceiling with latex tubing. The vertical and horizontal resonant frequencies for the latex-support system were 0.33 Hz which is lower by more than a factor of 10 a brick, rubber pad, and sandbox support. The linewidth of the heterodyne signal between the laser radiation stabilized to the cavity supported by the latex tubing and the cavity supported on the sandbox table was approximately 30 Hz or less (Fig. 5). Presumably the width of the heterodyne signal is dominated by the stability of the cavity on the sandbox table. Currently, we are working to verify this and working to build better cavities (higher finesse, increased stiffness and lower sensitivity to temperature and aging).

ACKNOWLEDGEMENTS

We gratefully acknowledge the support of the U.S. Office of Naval Research and the U.S. Air Force Office of Scientific Research. We thank A. DeMarchi, R. Drullinger, H. Robinson and M. Young for helpful comments on the manuscript.

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