Quantum Simulations of Spin Systems Using Trapped $^{88}Sr^{+}$ Ions

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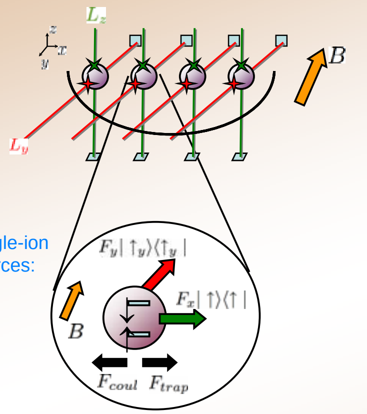
Quantum spin systems can be studied and simulated by trapped ions interacting with laser beams of different intensities, frequencies, and polarizations. Following Ref. [1] we present numerical results showing that, in certain limits, a magnetic field together with a state-dependent dipole force acting on the ions produces an effective many-spin evolution of the system in linear and micro-traps.

[1] D. Porras and J. I. Cirac, Phys. Rev. Lett. 92, 207901-1 (2004).

Experimental setting: Ions in macro- or micro-traps

Trapped Ions: -Real or simulated *B*-field;
-Laser beams generate dipole forces



Single-ion forces:

Interactions: $\boxed{H = H_m + H_{ph} + H_f}$

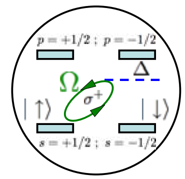
$$H_m = \sum_{j,\alpha} B^\alpha \sigma_j^\alpha \quad \text{magnetic field}$$

$$H_{ph} = \sum_{n,\alpha} \omega_n^\alpha a_n^{\alpha\dagger} a_n^\alpha \quad \text{vibrational motion}$$

$$H_f = -F_x \sum_j x_j |\uparrow\rangle\langle\uparrow|_j - F_y \sum_j y_j |\uparrow_y\rangle\langle\uparrow_y|_j \quad \text{dipole force}$$

$$\alpha_j = \sum_n \frac{V_{jn}^\alpha}{\sqrt{2m\omega_{\alpha,n}/\hbar}}(a_n^{\alpha\dagger} + a_n^\alpha) \quad \text{(ion position operator)}$$

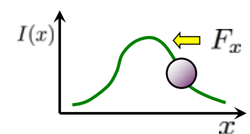
Dipole Force on $^{88}Sr^{+}$ Ions: detuned laser beams



$^{88}Sr^{+}$

$$H(x) \propto \begin{array}{c|cc} & |\uparrow\rangle\langle\uparrow| & |\uparrow\rangle\langle\downarrow| \\ & \begin{pmatrix} I(x)/4\Delta & 0 \\ 0 & 0 \end{pmatrix} & \\ & |\downarrow\rangle\langle\uparrow| & |\downarrow\rangle\langle\downarrow| \end{array}$$

Intensity Profile:



$$\rightarrow H(x) \approx H(x_0) + x\underbrace{\kappa \frac{\partial I}{\partial x}}_{F_x} |\uparrow\rangle\langle\uparrow|$$

Intensity gradient generates the force

(See K. Vant's et. Al. poster for details)

Transformation to a spin-1/2 system

$$S = \sum_{\alpha,j,n} \eta_{jn}^\alpha (1 + \tilde{\sigma}_j^\alpha)(a_n^{\alpha\dagger} - a_n^\alpha)\ ;\ \eta_{jn}^\alpha = F_\alpha \frac{V_{jn}^\alpha}{\hbar\omega_{\alpha,n}} \sqrt{\frac{\hbar}{2m\omega_{\alpha,n}}}$$

Unitary transformation maps *H* to spin-spin interactions (effective Hamiltonian H_S^{eff})

$$e^{-S} H e^{S} = H_{ph} + \underbrace{\frac{1}{2}\sum_{\alpha,i,j} J_{ij}^\alpha \tilde{\sigma}_i^\alpha \tilde{\sigma}_j^\alpha + \sum_{\alpha,j} \tilde{B}^\alpha \sigma_j^\alpha}_{H_S^{eff}} + H_E$$

Extra term: Spin-phonon coupling

H_S^{eff} ⇒ Effective interactions: Spin-1/2 model

Spin-spin interactions $\quad J_{ij}^\alpha = \sum_n \frac{F_\alpha^2}{m\omega_{\alpha,n}^2} V_{in}^\alpha V_{jn}^\alpha \quad ; \quad \tilde{B}^\alpha = B^\alpha + F_\alpha^2/(m\omega_\alpha^2)$

For small displacements:

$$\eta_{jn}^\alpha \ll 1$$

$$(H_E << H_S^{eff})$$

Heisenberg-like Models

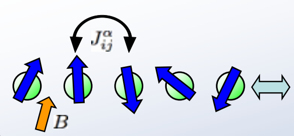
Two forces: F_x, F_y

$$H_S^{eff} = \frac{1}{2}\sum_{i,j}(J_{ij}^x \sigma_i^z \sigma_j^z + J_{ij}^y \sigma_i^y \sigma_j^y) + \sum_{j,\alpha} B^\alpha \sigma_j^\alpha$$

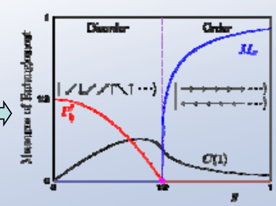
Ising-like Models

Single force: $F_y = 0\ ;\ B = B^x$

$$H_S^{eff} = \frac{1}{2}\sum_{i,j} J_{ij}^x \sigma_i^z \sigma_j^z + \sum_j B^x \sigma_j^x$$



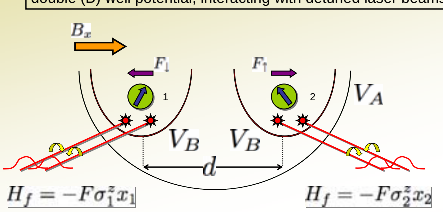
⇔ Quantum Phase Transitions (*N*>>1) ⇔



Two-Spin Simulations: linear and micro-traps

Ising Interactions ⇒ $H_{Ising} = J\sigma_1^z \sigma_2^z + B_x(\sigma_1^x + \sigma_2^x)$

Simulation of the dynamics of two ions in a single (A) and double (B) well potential, interacting with detuned laser beams.



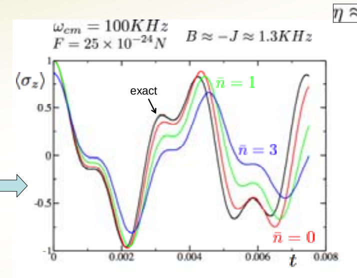
Two normal modes:
- Symmetric (center of mass mode): ω_{cm}
- Antisymmetric (breath mode): ω_{br}

$$\eta_i = \frac{F}{2\sqrt{m\omega_i^3 \hbar}} \ll 1$$

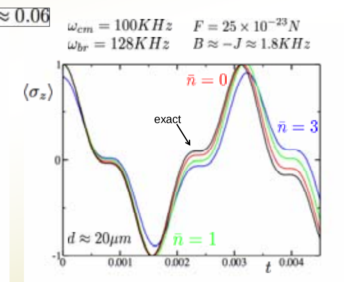
$$\boxed{H \approx H_{Ising} + H_{ph}}$$

Spin-spin coupling ⇒ $J = \frac{F^2}{2m}(\omega_{br}^{-2} - \omega_{cm}^{-2})$

$\eta \approx 0.06$



Ⓐ Linear trap



Ⓑ Micro-trap (See K. Vant's et. Al. poster for photos)

$\rho(0) = |\uparrow\uparrow\rangle\langle\uparrow\uparrow| \otimes \langle\bar{n}\rangle$

$\rho(t) = e^{-iHt}\rho(0)e^{iHt}$

Cooling to $\bar{n} \approx 1$ is required to achieve the quantum simulation with small error.