Progress toward making spin squeezed states with ions in a Penning-Malmberg trap

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Prospects for entangling ions in a Penning trap

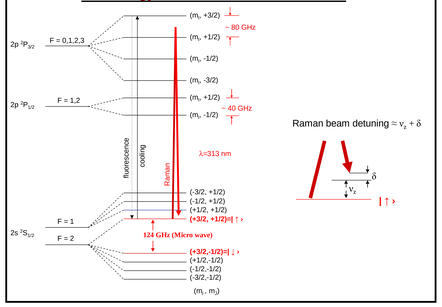
Why entangle ions in a Penning trap?

- large numbers of ions
- long relaxation times
- low heating rate (?)
- study of the decoherence of large N spin squeezed states
- potentially useful for spectroscopy

N ions, $\vec{s}_i = \vec{\sigma}_i/2$, composite spin $\vec{J} = \sum_{i=1}^{N} \vec{s}_i$

Ramsey squeezing parameter $\xi_R \equiv \frac{\langle \Delta J_\perp \rangle}{|\langle \vec{J} \rangle|}\sqrt{N}$

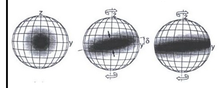
$^9Be^+$ energy level structure with B= 4.5 T



Spin squeezing

Wineland et al., PRA 46, 3554 (93)
Kitagawa & Ueda, PRA 47, 5138 (93)

one axis twisting



start in $|J, M_J\rangle = |\frac{N}{2}, \frac{N}{2}\rangle$
rotate $\frac{\pi}{2}$ radians about y
apply $\exp(i\chi J_z{}^2 t) = \exp(i\theta J_z{}^2)$

$\langle J_x \rangle = \frac{N}{2}\cos^{N-1}(\theta)$
$\langle \Delta J_\perp{}^2 \rangle_{min} = \frac{N}{4}\{1 + \frac{N-1}{4}[A - \sqrt{A^2+B^2}]\}$
$A = 1 - \cos^{N-2}(2\theta),\ B = 4\sin(\theta)\cos^{N-2}(\theta)$

$\langle \Delta J_\perp{}^2 \rangle_{min}, \xi_R = \frac{\langle \Delta J_\perp \rangle}{J_z}\sqrt{N}$ are minimized at short times $\theta_{opt} \sim \frac{0.86}{N^{2/3}}\frac{\pi}{2}$

Example: N=1000 → θ_{opt} ~ 0.0086·(π/2) , ξ_R~ 0.11
N= 100 → θ_{opt} ~ 0.04·(π/2) , ξ_R ~ 0.25

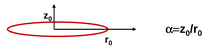
Mode spacing for single plane plasmas

(Weimer et al., PRA 49, 3842)

- **single plane condition:** $\sigma(0)\left(\frac{4\pi e^2}{m\omega_z{}^2}\right) < 1.10$ where $\sigma(r) = 2\varepsilon_0 n_0\sqrt{1-(r/r_0)^2}$
(Dubin, PRL 71, 2753)
⇒ plasma radius $r_0 > \left(\frac{3N}{0.8\pi}\right)^{1/2}\left(\frac{3e^2}{m\omega_z{}^2}\right)^{1/3}$ ← independent of B-field

- consider axially symmetric drumhead modes

$\omega_{1,0} = \omega_z$
$\omega_{3,0} = \omega_z\{1-5\pi\alpha/16\}$



$\alpha = z_0/r_0$

mode spacing $\sim \omega_z\alpha$
$\sim 0.8\frac{\omega_z}{\sqrt{N}}$ ← for spherical plasma mode spacing ~ 1/N

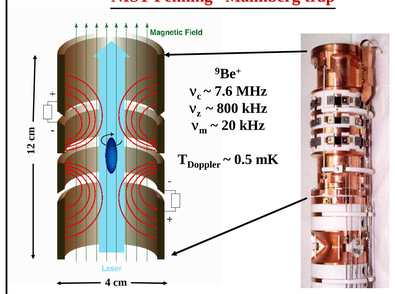
$\alpha \sim \frac{3Ne^2}{m\omega_z{}^2 R_p{}^3}$

assume $\omega_z/(2\pi)$ ~ 1 kHz

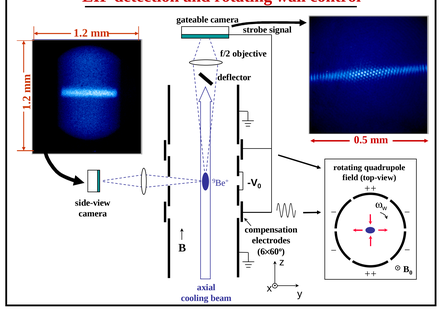
N	$0.8\omega_z/(2\pi\sqrt{N})$
10	250 kHz
100	80 kHz
1000	25 kHz
10000	8 kHz

probably want δ << mode spacing

NIST Penning –Malmberg trap



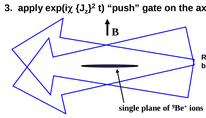
LIF detection and rotating wall control



Entangling scheme: NIST two ion-qubit phase gate

[Leibfried et al., Nature 422, 412 (2003)]

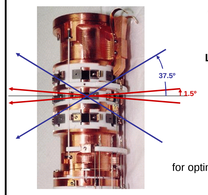
1. **prepare** $|\uparrow\uparrow\uparrow\cdots\uparrow\rangle = |J=\frac{N}{2}, M_J=\frac{N}{2}\rangle$. **$T_{motional}$ ~ 1 mK**
2. **π/2 pulse of 124 GHz microwaves** $|J=\frac{N}{2}, M_J=\frac{N}{2}\rangle \rightarrow \sum_{M_J=-\frac{N}{2}}^{\frac{N}{2}} c(N, M_J)|J, M_J\rangle$
3. **apply exp(iχ $\{J_z\}^2$ t) "push" gate on the axial center-of-mass mode of a single ion plane**



$\chi = \frac{(\eta\Omega)^2}{\delta}$
$t = m\frac{2\pi}{\delta},\ m = 1, 2, 3, \ldots$

4. **do Ramsey spectroscopy to extract ξ_R**

Experimental parameters



Assume: ν_z = 1 MHz, T = 1 mK

$\eta_{com} = \sqrt{\frac{\hbar}{2m\omega_z}}\frac{1}{\sqrt{N}}2k\sin\theta$

Lamb Dicke limit $\eta_{com}\sqrt{\bar{n}+1} \ll 1$
$\bar{n} \simeq \frac{k_B T}{\hbar\omega_z} \approx 20$

5 mW beams, single plane with radius R_p, require $\exp(-2R_p{}^2/w_0{}^2) > 0.9$
⇒ laser waist $w_0 = 4.36\ R_p$

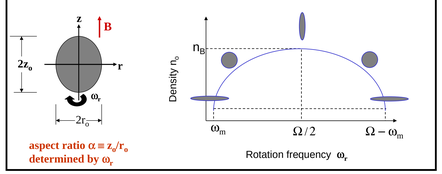
for optimum squeezing $\frac{\eta_{com}^2\Omega^2}{\delta}\frac{2\pi}{\delta}m = \frac{0.86}{N^{2/3}}\frac{\pi}{2}$

N, θ	R_p	Ω/(2π)	η_{com}	$\eta_{com}\sqrt{\bar{n}+1}$	η_{com} Ω/(2π)	m	interaction time for optimum squeezing
10, 1.5°	35 μm	60 kHz	0.008	0.036	480 Hz	0.2	~ 1 ms
10^2, 37.5°	112 μm	6 kHz	0.058	0.27	340 Hz	0.3	~ 1 ms
10^3, 37.5°	352 μm	600 Hz	0.018	0.084	11 Hz	17.5	18 ms
10^4, 37.5°	1120 μm	60 Hz	0.006	0.027	0.35 Hz	3720	3.7 s

Equilibrium plasma properties

Dubin and O'Neil, RMP 71, 87 (99)

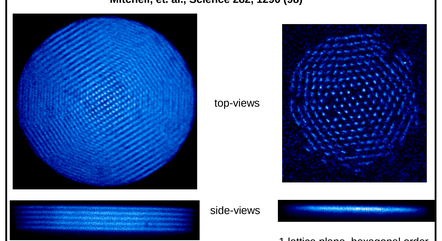
- **thermal equilibrium ⇒ rigid rotation ω_r**
- **T ~ 0 ⇒ constant plasma density, $n_0 = 2\varepsilon_0 m\omega_z(\Omega-\omega_z)/q^2$, Ω = cyclotron frequency**
- **quadratic trap potential ⇒ plasma shape is a spheroid**



aspect ratio $\alpha \equiv z_0/r_0$ determined by ω_r

Planar structures at low rotation frequencies

Mitchell, et. al., Science 282, 1290 (98)



5 lattice planes, bcc-like order

1 lattice plane, hexagonal order

1st step – need to be able to perform projection-noise limited spectroscopy on the electron spin flip

projection noise vs shot noise at "side"

$\sigma_{proj} = K\frac{1}{2}\sqrt{N},\quad \sigma_{shot} = \sqrt{K\frac{N}{2}}$

$\sigma_{proj} > 10\sigma_{shot} \Rightarrow K\frac{1}{2}\sqrt{N} > 10\sqrt{K\frac{N}{2}}$

$K > 200$

K = # of photons detected for each bright state ion

at high magnetic field (4.5 T), optical pumping slow ~ 5 s

f/2 10^7 photons/s • 0.015 • 0.3 • 0.8 • 1 s = 36000 detected photons/bright ion
f/5 10^7 photons/s • 0.0025 • 0.3 • 0.8 • 1 s = 6000

saturation count rate; solid angle; QE; lens transmission

Spontaneous emission

for Be^+ with a Raman detuning of 40 GHz, γ ~ Ω·10^{-3}

$\langle \vec{J} \rangle = |\langle \vec{J} \rangle|\hat{z},\quad J_x = \sum_{i=1}^{N} s_{x,i},\quad \langle J_x \rangle = e^{-\gamma t}\langle J_x(0) \rangle$

$\langle J_x{}^2(t) \rangle = \langle \sum_{i,j=1}^{N} s_{x,i}s_{x,j} \rangle = \frac{N}{4} + e^{-2\gamma t}\sum_{i\neq j}\langle s_{x,i}(0)s_{x,j}(0) \rangle$

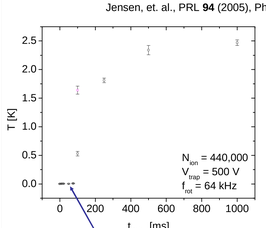
$= \frac{N}{4} + e^{-2\gamma t}[\langle J_x{}^2(0) \rangle - \frac{N}{4}]$

$\xi_R(t) = \sqrt{\frac{N^2}{4|\langle J_z(0) \rangle|^2}(1-e^{-2\gamma t}) + e^{-2\gamma t}\xi_R{}^2(0)}$ ← spin squeezing sensitive to correlations between pairs of spins

N	γ = Ω × 10^{-3}	t	$\xi_R(0)=\xi(\theta_{opt})$	$\xi_R(t)$
10	2π× 60 s^{-1}	10^{-3} s	0.56	0.89
100	2π× 6 s^{-1}	10^{-3} s	0.25	0.38
1000	2π× 0.6 s^{-1}	20× 10^{-3} s	0.11	0.43
10000	2π× 0.06 s^{-1}	3.7 s	0.05	1.07

Ion heating rate

Jensen, et. al., PRL **94** (2005), Phy. Rev. A **70** (2004)



How much time is there for entangling the ions?

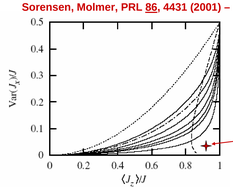
10 ms ? longer ??

Slow heating at short times: 50-100 mK/s due to residual gas collisions

Possible studies

1. use Ramsey spectroscopy to measure $|\langle \vec{J} \rangle|$ and $\Delta J_{\perp,min}$ calculate ξ_R
2. measure the decoherence - measure how $|\langle \vec{J} \rangle|$ and $\Delta J_{\perp,min}$ evolve
3. how does the decoherence scale with N and the "depth" of the squeezing

Sorensen, Molmer, PRL 86, 4431 (2001) – measure "depth" of entanglement

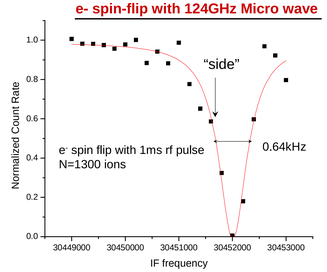


maximal squeezing curves for J= ½, 1, 3/2, 2, 3, 4, 5, and 10

if a state lies below a curve for a particular J, it is a 2J-particle entangled state

$e^{i\chi J_z{}^2 t}$ result for N = 100 and $|\langle \vec{J} \rangle|$~0.92

e- spin-flip with 124GHz Micro wave



$\nu_{\mu w}$=124,048.248MHz+IF Frequency