Passive Cooling of a Micromechanical Oscillator with a Resonant Electric Circuit

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We cool the fundamental mode of a miniature cantilever by capacitively coupling it to a driven rf resonant circuit. Cooling results from the rf capacitive force, which is phase shifted relative to the cantilever motion. We demonstrate the technique by cooling a 7 kHz cantilever from room temperature to 45 K, obtaining reasonable agreement with a model for the cooling, damping, and frequency shift. Extending the method to higher frequencies in a cryogenic system could enable ground state cooling and may prove simpler than related optical experiments in a low temperature apparatus.

Stimulated by the early work of Braginsky and collaborators [1,2], the quantum-limited measurement and control of mechanical oscillators continues to be a subject of great interest. If one can cool to the ground state of the oscillator, the generation of nonclassical states of motion also becomes feasible. For an atom bound in a harmonic well, laser cooling in a room-temperature apparatus can cool the modes of mechanical motion to a level with mean occupation numbers \( n < 0.1 \) for oscillation frequencies \( \sim 1–10 \text{ MHz} \) [3,4]. This has made it possible to generate nonclassical mechanical oscillator states such as squeezed, Fock [5], multiparticle entangled [6], and (in principle) arbitrary superposition states [7].

For more macroscopic systems, smaller and smaller micromechanical resonators have approached the quantum limit through thermal contact with a cryogenic bath (for a summary, see [8]). Small mechanical resonators, having low-order mode frequencies of \( 10–1000 \text{ MHz} \), can come close to the quantum regime at low temperature (<1 K), and mean occupation numbers of approximately 25 have been achieved [9]. Cooling of macroscopic mechanical oscillators also has been achieved with optical forces. The requisite damping can be implemented by use of active external electronics to control the applied force [10–13] (see also [14]). Passive feedback cooling has been realized in which a mirror attached to a mechanical oscillator forms an optical cavity with another stationary mirror. For appropriate tuning of radiation incident on the cavity, a delay in the optical force on the oscillator as it moves gives cooling. This delay can result from a photothermal effect [15,16] or from the stored energy response time of the cavity [17–19]. Closely related passive cooling has been reported in [9,20].

We demonstrate a similar cooling mechanism where the damping force is the electric force between capacitor plates [21] that here contribute to a resonant rf circuit [2,22]. This approach has potential practical advantages over optical schemes: eliminating optical components simplifies fabrication and integration into a cryogenic system, and the rf circuit could be incorporated on-chip with the mechanical oscillator.

A conducting cantilever of mass density \( \rho \) is fixed at one end [Fig. 1(a)]. One face is placed a distance \( d \) from a rigidly mounted plate of area \( w \times h \), forming a parallel-plate capacitor \( C_c = \varepsilon_0 w h / d \), where \( \varepsilon_0 \) is the vacuum dielectric constant. An inductor \( L_0 \) and capacitor \( C_0 \) in parallel with \( C_c \) form a resonant rf circuit with frequency \( \Omega_0 = 1 / \sqrt{L_0 (C_0 + C_c)} \) and with losses represented by resistance \( R_0 \). We assume \( Q \ll 1 \), where \( Q \ll \Omega_0 / \gamma \) and \( \gamma \) is the damping rate.

We consider the lowest-order flexural mode of the cantilever, where the free end oscillates in the \( \hat{x} \) direction [vertical in Fig. 1(a)] with angular frequency \( \omega_c \ll \gamma \).

FIG. 1. Schematics of the cantilever cooling and detection electronics. (a) Cantilever and associated rf circuitry. (b) Motional detection electronics. Near \( \omega_c \), the rf circuit looks like a short to ground as shown. (c) Equivalent circuit for the cantilever and detection electronics near \( \omega \approx \omega_c \).
We take $x$ to be the displacement at the end of the cantilever, so the displacement as a function of (horizontal) position $z$ along the length of the cantilever is given by $x(z) = f(z)x$, where $f(z)$ is the mode function (see, e.g., [23]). Small displacements due to a force $F$ can be described by the equation of motion,

$$m \ddot{x} + m\Gamma \dot{x} + m\omega^2x = F,$$

where $\Gamma$ is the cantilever damping rate and $m$ its effective mass, given by $\rho \varepsilon_c w h s$, where $\varepsilon_c = \frac{1}{h} \int_0^h f(z)^2 dz = 0.250$ for a rectangular beam.

For simplicity, first assume $h \ll h_c$, so that the force is concentrated at the end of the cantilever. If a potential $V$ is applied across $C_c$, the capacitor plates experience a mutual attractive force $F = \varepsilon_0 w h V^2/(2d) = C_c V^2/(2d)$. Consider that $V$ is an applied rf potential $V_{rf}$, so the force is $F = \varepsilon_0 w h V_{rf}^2/(2d)$.

Where $\omega_c \ll \omega_0$, the force for frequencies near $\omega_c$ can be approximated by the time-averaged rf force

$$F_{rf} = \frac{C_c (V_{rf})^2}{2d} = \frac{C_c V_{rf}^2}{4d} = \frac{\varepsilon_0 w h V_{rf}^2}{4d^2},$$

where, for a fixed input rf power, $V_{rf}$ will depend on $\Delta \Omega \equiv \Omega_0 - \Omega_{rf}$, according to

$$\frac{V_{rf}^2}{V_{max}^2} = \frac{1}{1 + [2Q_{rf} \Delta \Omega / \Omega_0]^2} \equiv L(\Delta \Omega).$$

As the cantilever oscillates, its motion modulates the capacitance of the rf circuit thereby modulating $\Omega_0$. As $\Omega_0$ is modulated relative to $\Omega_{rf}$, so too is the rf potential across the cantilepce, according to Eq. (3). The associated modulated force shifts the cantilever’s resonant frequency. Because of the finite response time of the rf circuit, there is a phase lag in the force relative to the motion. For $\Delta \Omega \gg 0$ the phase lag leads to a force component that opposes the cantilever velocity, leading to damping. If this damping is achieved without adding too much force noise then it cools the cantilever.

The average force due to applied potentials displaces the equilibrium position $d_0$ of the cantilever. We assume this displacement is small and is absorbed into the definition of $d_0$, writing $d = d_0 - x$, where $x \ll d_0$. Following [2] or [22] we find $\omega^2_c \rightarrow \omega^2_c (1 - \kappa)$ and $\Gamma \rightarrow \Gamma + \Gamma'$, with

$$\kappa = \frac{C_c V_{max}^2 L(\Delta \Omega)}{2 m \omega_c d_0^2} \left[ \frac{\xi''}{\gamma} + \frac{2(\xi')^2 Q_{rf} \Delta \Omega L(\Delta \Omega)}{\gamma} \times \frac{C_c}{C_c + C_0} \right],$$

$$\Gamma' = \frac{Q_{rf} V_{max}^2 C_c^2}{m \omega_c d_0^2 (C_c + C_0)} \frac{(\xi')^2 \Delta \Omega L(\Delta \Omega)}{\gamma} \sin \phi,$$

where $\xi' = \frac{1}{h} \int_0^h f(z)^2 dz$ and $\xi'' = \frac{1}{h} \int_0^h f(z)^2 dz$ are geometrical factors required when $h \ll h_c$ is not satisfied. The phase $\phi$ is equal to $\omega_c \tau$, where $\tau = 4L(\Delta \Omega)/\gamma$ is the response time of the rf circuit [25]. For $\Delta \Omega > 0$, $\Gamma'$ gives increased damping. For $\Delta \Omega = \gamma/2$ and $h \ll h_c$ ($\xi' = \xi'' = 1$), we obtain the expressions of [22]. For our experiment, $\gamma = h_c$, $\xi' = 0.392$, and $\xi'' = 0.250$.

We detect the cantilever’s motion by biasing it with a static potential $V_h$ through resistor $R_b$ as shown in Fig. 1(b), where $R_a$, $C_a$, $i_a$, and $e_a$ represent the equivalent input resistance, capacitance, current noise, and voltage noise, respectively, of the detection amplifier. We make $R_a$ and $R_b$ large to minimize their contribution to the current noise $i_a$. We assume $C_i \gg (C_a + C_c)$ and $\omega_c R(C_a + C_c) \gg 1$, where $1/R = 1/R_b + 1/R_a$. As the cantilever moves, thereby changing $C_c$, it creates a varying potential that is detected by the amplifier.

The (charged) cantilever can be represented by the series electrical circuit in Fig. 1(c). From Eq. (2) and following [26], the equivalent inductance is given by $L_{eq} = m d_0^2 / (q_c \xi')^2$, where $q_c$ is the average charge on the cantilever. From $L_{eq}$, $\omega_c$, and $\Gamma$, we can then determine $C_{eq} = 1/(\omega_c^2 L_{eq})$ and $R_{eq} = L_{eq} \Gamma$. Additional damping due to the rf force is represented by $R_{rf} = L_{eq} \Gamma'$. For frequencies $\omega = \omega_c$, the parallel combination of $R_b$, $C_a$, and $R_a$ can also be expressed instead as the Thévenin equivalent $R_c$, $C_c$ circuit in Fig. 1(c). The amplifier’s current noise $i_a$ is now represented as $e_a(R_b)$. The intrinsic thermal noise of the cantilever is characterized by a noise voltage $e_n(R_{eq})$ having spectral density $4k_b T R_{eq}$, where $k_b$ is Boltzmann’s constant and $T_c$ is the cantilever temperature.

We must also consider noise from the rf circuit. In Eq. (2), we replace $V_{rf}$ with $V_{rf} + v_{rf}(t)$, where $v_{rf}(t)$ is the noise across the cantilever capacitance $C_c$ due to resistance in the rf circuit and noise injected from the rf source. The cantilever is affected by amplitude noise $v_{n}(t)$ at frequencies near $\Omega_{rf} \pm \omega_c$, because cross terms in Eq. (2) give rise to random forces at the cantilever frequency. This noise force can be represented by $e_n(t)$ in the equivalent circuit. The noise terms sum to $e_n^2 = e_n^2(R_{eq}) + e_n^2(R_c) + e_n^2(R_b)$ ($e_n$ does not drive the cantilever), which gives a cantilever effective temperature

$$T_{eff} = \frac{e_n^2}{4 k_b R_{eq} + R_f + R_b}.$$

Our cantilever has nominal dimensions $h_c = 1.5$ mm, $s = 14 \mu m$ [27], and $w = 200 \mu m$, created by etching through a $p^{++}$-doped ($\sim 0.001 \Omega$ cm), 200 $\mu m$ thick silicon wafer with a standard Bosch reactive-ion-etching process. Its resonant frequency and quality factor are $\omega_c/(2\pi) = 7$ kHz and $Q = 20,000$. The cantilever is separated by $d_0 = 16 \mu m$ [27] from a nearby doped silicon rf electrode, forming capacitance $C_c$. The sample is enclosed in a vacuum chamber with pressure less than $10^{-5}$ Pa. The rf electrode is connected via a vacuum feedthrough to a quarter-wave resonant cavity with $L_0 = 330(30)$ nH and with loaded quality factor $Q_{rf} = 234(8)$ at $\Omega_{rf}/(2\pi) = 100$ MHz when impedance matched to the source. The cantilever is connected by a short length of coaxial cable and blocking capacitor $C_i = 4 \text{ nF}$ to a low-noise junction.
field-effect transistor amplifier [see Fig. 1(b)]. We have $C_a = 48(1)$ pF, with $R_a = R_b = 1$ GΩ. We use $V_{pp} = -50$ V, which gives a measured 2.5 μm static deflection at the cantilever end.

We temporarily lowered $R_a$ to 600 kΩ = $1/(\omega_e C_a)$, in which case the cantilever noise spectrum strongly distorts from a Lorentzian line shape (not shown), and it becomes straightforward to extract the equivalent circuit parameters of Fig. 1(c). We find $L_{eq} = 27700(600)$ H. To lowest order in rf power this equivalent inductance remains constant, so it is straightforward to extract the equivalent circuit parameters at the cantilever end.

For $R_a = 1$ GΩ we measure $e_a = 1.5$ nV/√Hz and $i_a = 16$ fA/√Hz. Figure 2 shows a series of thermal spectra acquired with this value of $R_a$ at different values of rf power $P_{rf}$ but at constant detuning $\Delta \Omega = 2\pi \times 90$ kHz = $0.21\gamma$. Both the lowering and the broadening of the spectra with increasing $P_{rf}$ are evident, in accordance with Eq. (5). Here, the effective temperature is very nearly proportional to the area under the curves, although there is a slight asymmetric distortion from a Lorentzian line shape, fully accounted for by the equivalent circuit model. The center frequency of each spectrum also shifts to lower frequencies for increasing $P_{rf}$, as predicted by Eq. (4) and the definition of $\omega_e$. After calibrating the gain of the amplifier, we extract $e_a^2$ for each spectrum from a fit to the model of Fig. 1(c). The absolute effective temperature is then given by Eq. (6).

Equations (5) and (6) predict that the cantilever’s effective temperature should fall with increasing $P_{rf}$, as demonstrated by the data in Fig. 3 for low power. With no rf applied we find $T_{eff} = 310(20)$ K. The coldest spectrum corresponds to a temperature of 45(2) K, a factor of 6.9 reduction. The minimum temperature appears to be limited by AM noise from our rf source. This noise power at $\Omega_{rf} \pm \omega_c$ is constant relative to the carrier, leading to a noise power at $\omega_c$ given by $e_a(rf)^2 \approx P_{rf}^2$. We fit the residual noise $e_a(rf)^2$ to a quadratic in $P_{rf}$, giving the dashed line temperature prediction in Fig. 3. From this fit we determine that the AM noise of our source is = $170$ dBc/Hz, reasonably consistent with the value ($-167$ dBc/Hz) measured by spectrum analysis.

The inset shows the cantilever damping rate $\Gamma$ versus $P_{rf}$. The slope is $\Gamma/P_{rf} = 5450(70)$ Hz/W, slightly higher than the value 3970 Hz/W calculated from Eq. (5) and the nominal cantilever dimensions. The nonlinearity in $\Gamma/P_{rf}$ at higher powers is consistent with the cantilever being pulled toward the rf electrode. We have numerically simulated this effect and find reasonable agreement. The variation of $\kappa$ with $P_{rf}$ (not shown) is also linear, with a slope $\kappa/P_{rf} = 7.64(8)$ W$^{-1}$, compared with the value 3.45 W$^{-1}$ calculated from Eq. (4).

Although $\Gamma/P_{rf}$ and $\kappa/P_{rf}$ differ from their predicted values, this disagreement is not unexpected considering the relatively large variations in dimensions $d_0$ and $s$ [27]. Another indication of these uncertainties is that optical measurements of the static deflection of the cantilever along its length disagree with predictions based on a constant cantilever cross section. This will lead to deviations from our calculated values of $\xi', \xi''$, and $\xi''$. However, we stress that these deviations should not give significant errors in our measured values of $L_{eq}$, $R_{eq}$, and therefore our determination of $T_{eff}$.

To further test the model, we examine $\Gamma$ and $\omega_c$ as a function of $\Delta \Omega$ (Fig. 4). For large detunings $\Delta \Omega$, $\Gamma$ asymptotically approaches the value obtained in Fig. 3 for $P_{rf} = 0$. Near $\Delta \Omega = 0$, $\omega_c$ is generally shifted to a lower value, while $\Gamma$ is either enhanced or suppressed,
For example, if \( T_c \) is not surprising for the reasons mentioned above. Solid lines are fits to Eqs. (4) and (5).

According to the sign of \( \Delta \Omega \), data cannot be obtained for \( \Delta \Omega < 0 \) when the rf power level is sufficient to drive the cantilever into instability (\( \Gamma < 0 \)). The solid line fits show good agreement with the predicted behavior. From these fits we extract \( C_r = 0.09 \) pF, lower than the value 0.17 pF obtained from the physical dimensions. This disagreement is not surprising for the reasons mentioned above.

Some experiments using optical forces have observed strong effects from the laser power absorbed in the cantilever mirror. A conservative estimate of the rf power dissipated in our cantilever gives a temperature rise of less than 1 K at the highest power, so these effects should not be significant.

Although rather modest cooling is obtained here, the basic method could eventually provide ground state cooling. For this we must achieve the resolved sideband limit, where \( \omega_c > \gamma \) [25,28], and the cooling would be very similar to the atomic case [3,4]. To insure a mean quantum number \( n \) less than one, the heating rate from the ground state \( n_{\text{heat}} = \Gamma k_B T_c/(\hbar \omega_c) \) must be less than the cooling rate for \( n = 1 \rightarrow 0 \). The cooling rate \( n_{\text{cool}} \) can be estimated by noting that each absorbed photon on the lower sideband (at the applied rf frequency \( \Omega_0 - \omega_c \)) is accompanied by reradiation on the rf “carrier” at \( \Omega_0 \). If we assume the lower sideband is saturated for \( n = 1 \), \( n_{\text{cool}} = \gamma/2 \). Hence we require \( R \equiv n_{\text{heat}}/n_{\text{cool}} = 2 k_B T_c Q_c/(\hbar \Omega_0 Q_c) \ll 1 \).

For example, if \( T_c = 0.1 \) K, \( \Omega_0/(2\pi) = 20 \) GHz, \( Q_c = 5000 \) (e.g., a stripline), and \( Q_c = 20000 \) we have \( R \approx 0.05 \). For resolved sidebands, we require \( \omega_c/(2\pi) > 4 \) MHz.

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[24] This expression for \( d \) neglects the curvature of the flexural mode and is strictly true only for \( h \ll h_c \) and \( d_0 \ll w, h \).
[27] The uncertainties for these numbers are a few micrometers, stemming from spatial nonuniformity in our etch process.