A femtosecond-laser-based optical clockwork with instability $\leq 6.3 \times 10^{-16}$ in 1 s*

Scott. A. Diddams and L. Hollberg

National Institute of Standards and Technology,

325 Broadway, Boulder, CO 80305

L.-S. Ma[†]

JILA, University of Colorado, Boulder, CO 80309

L. Robertsson

BIPM, Pavillon de Breteuil, 92312 Sevres, France

*Submitted to Optics Letters.

Abstract

Two octave-spanning optical-frequency combs (750 MHz comb spacing), are phase-locked to a common continuous-wave laser diode. The measured instability of the heterodyne beat between the two combs demonstrates that the intrinsic noise of a comb is $\leq 6.3 \times 10^{-16}$ in 1 s of averaging across the ~ 300 THz bandwidth. Furthermore, the average frequencies of the elements of the two combs are found to agree within an uncertainty of 4×10^{-17} across the entire octave. This work demonstrates the possibility to transfer the stability and accuracy of the best current optical standards to $\sim 500,000$ individual oscillators across the visible and near-infrared spectrum.

The introduction of femtosecond laser technology into the field of precision optical-frequency metrology[1,2] has rapidly culminated in a single-step phase coherent connection between emerging optical frequency standards and the cesium microwave standard on which the SI second is based.[3,4] Using femtosecond lasers in combination with microstructure optical fibers, it is now possible to produce an octave-spanning spectrum in the visible and near-infrared. Because it originates from a mode-locked laser, this octave-spanning spectrum consists of discrete modes, each of which has frequency $f_n = nf_r + f_o$, where f_r is the pulse repetition frequency, n is an integer and f_o is an offset common to all modes. When the frequencies f_r and f_o are phase-locked to a microwave standard of given uncertainty, every element of the optical comb can be known with the same uncertainty. The octave-spanning comb then operates as a clockwork that permits a single-step, phase-coherent multiplication of the microwave standard up to the optical domain. Tests of the stability and accuracy of the clockwork with a microwave reference have been performed.[5–8]

Of greater interest and ultimate importance is the use of the femtosecond-laser-based clockwork to transfer the stability and accuracy of an optical standard down to a countable microwave frequency, as required for building an optical atomic clock. In this case, the precision of the clockwork must not degrade the exceptionally low fractional frequency instability of current optical standards, which can be $\leq 4 \times 10^{-15}$ in 1 s of averaging.[9,10] We have recently demonstrated an optical atomic clock based on this concept with instability $\leq 7 \times 10^{-15}$ in 1 s.[11] However, this level of instability was limited by the present configuration of our Ca standard, so that potential limitations due to the clockwork remained untested. In this Letter, we establish an upper limit of 6.3×10^{-16} for the 1 s instability of the femtosecond-laser-based clockwork. This is measured in the optical heterodyne beat between two independent octave-spanning frequency combs that have the same value of f_r but different values of f_o. We further verify that the frequencies of elements of each comb across an entire octave are equal to their expected phase-locked values with an uncertainty of 4×10^{-17}. This represents a factor of ~ 12 improvement over the best previous test of the uncertainty of the nonlinearly generated comb frequencies,[5] and conclusively demonstrates that the stability and accuracy of the highest quality optical standards can be faithfully transferred to hundreds of thousands of individual comb elements across the entire visible and near-infrared spectrum. This is critical to the development of optical clocks because the frequency spacing (f_r) between the modes is the clock's countable output, which should then possess the same stability and accuracy as the optical standard.

The concept of our measurement is to phase-coherently lock all elements of two octave-spanning frequency combs to a CW reference laser. Subsequently, we measure and analyze the heterodyne beats between the two combs in different spectral regions to determine how precisely they track the reference laser [Fig. 1]. Since the noise of the CW laser is common to both combs, we can evaluate the precision of the various phase-lock loops (PLL's), as well as the stability and frequency accuracy of the comb elements that are nonlinearly generated in a microstructure optical fiber. More thorough descriptions of the generation and control of the frequency comb can be found in the references,[8,11] so here we present only the most relevant details. Each frequency comb is generated by coupling approximately 250 mW of the output of a mode-locked Ti:sapphire laser ($f_r = 750$ MHz)[12] into a 20 cm piece of microstructure optical fiber.[13] The spectrum out of the fiber spans the octave from 500 nm to > 1100 nm. We use the self-referencing technique[4,5] to determine f_o for each comb by frequency doubling the infrared components and heterodyning them with the visible components. We then phase-lock f_o to a stable radio frequency synthesized from a hydrogen maser (instability $\sim 2 \times 10^{-13}$ in 1 s), using the pump laser's power as the actuator.[5] This phase-locked optical beat is monitored with a high resolution counter, and the typical standard deviation in 1 s is ≤ 25 mHz ($\leq 5 \times 10^{-17}$). With f_o fixed in this manner, we control the comb's other degree of freedom (f_r) by measuring and phase-locking the heterodyne beat (f_b) between one element of the comb at 456 THz and a cavity-stabilized diode laser. In this case, a piezo-mounted mirror is used as the actuator. Because the comb elements from the femtosecond laser already have a well-defined phase relationship, phase-locking one mode to the diode laser in principle phase-locks all the modes to the diode laser. Indeed, the data presented here verifies this concept to a high degree of precision. Again, we count the phase-locked optical beat and find a typical standard deviation of ≤ 100 mHz in 1 s. This data alone implies that every mode of the octave-spanning comb tracks the diode laser with a relative uncertainty of $\leq 2 \times 10^{-16}$. We have made no attempt to orthogonalize the control of f_o and

f_r to reach this level, although doing so might improve the performance.[14]

Counting of the phase-locked beats as just described directly verifies the stability of the comb element at only the 456 THz frequency of the diode laser. A much more rigorous test involves the comparison of the two combs across their octave spans. To accomplish this, we offset the combs by an amount $\Delta f_o = f_{o1} - f_{o2} = 120$ MHz in the PLL's, while leaving f_r the same for both combs. We then spatially and temporally overlap portions of the beams from each system on a photodiode in order measure the heterodyne beat between the two combs. We set the temporal overlap by adjusting the phase-locked value of f_b in one system such that f_r differs slightly from that of the second system. With a fast oscilloscope, we monitor the arrival times of the two pulse trains as they come into coincidence on the photodiode. When exact coincidence is achieved, a strong beat at $\Delta f_o = 120$ MHz (S/N $\geq$ 40 dB in 300 kHz bandwidth) is observed and the phase-locked value of f_b is re-set so that f_r is again equal in both systems. This beat is bandpass filtered (6 MHz bandwidth), amplified and counted.

Using optical filters in conjunction with Si, GaAs and InGaAs detectors, we have measured the absolute value of Δf_o and its instability at 550 THz, 350 THz, and 275 THz. The results are summarized in Table 1. The Allan deviation (a measure of fractional frequency instability) computed from the counter readings of Δf_o at 550 THz is shown in Fig. 2. We observe the $\tau^{-1/2}$ dependence (square points) when the Allan deviation for $\tau > 1$ s is computed from the juxtaposition of a series of 1 s samples. However, when we compute the Allan deviation from data acquired with counter gate times of 3 and 10 s, we see a dependence closer to τ^{-1}. The decrease in slope from τ^{-1} to $\tau^{-1/2}$ is expected for white phase noise, where the rms phase fluctuation is constant in time.[15] We further find that the 1 s Allan deviation differs by no more than 30 % for the three measurement points across the octave. Since both combs contribute to the Allan deviation of Δf_o, we can assume that the fluctuations in a single comb are less by a factor of $\sqrt{2}$. Taking the Allan deviation at 550 THz, we then establish an upper limit of 6.3×10^{-16} for the 1-second instability of the femtosecond-laser-based clockwork. However, it is very likely that this upper limit is not due to the comb itself, but is instead simply a measurement of the uncontrolled fluctuations in path length between the two laser systems. For example, fluctuations of a few hundred nanometers on a 1 s time scale due to vibrating mechanics or air currents would lead to a fractional instability of $\sim 7 \times 10^{-16}$.

The offset of the measured value of Δf_o from the expected 120 MHz provides information about possible frequency errors that might occur in the nonlinear generation of the octave-spanning comb. This is a particularly important point, since this clockwork needs to ultimately be capable of supporting future optical standards with projected fractional frequency uncertainties in the 10^{-18} range.[16] The best previous test[5] of the actual frequencies of the elements an octave-spanning comb demonstrated an upper limit uncertainty of 5×10^{-16}. Tests of the uniformity of femtosecond-laser-based frequency combs have shown remarkable uncertainties as low as 3×10^{-18}; however, those tests did not control f_o and employed spectra broadened to only 44 THz in standard silica fiber.[17] At each optical frequency we made four measurement sets of the offset of Δf_o from 120 MHz (Fig. 3). The error bars on each point are indicative of the gate time and the number of counter readings in the measurement set. The largest offset is found at 350 THz, where the average offset is 19 ± 11 mHz, or fractionally $(5.6 \pm 3.0) \times 10^{-17}$. A shift of this order could be due to the thermal expansion of the optical table resulting from a temperature change of one degree in the laboratory over an hour. The weighted average of the three values of Table 1 provides an offset from 120 MHz of 0.14 mHz with an uncertainty of $\pm$ 5 mHz (fractionally $0.04 \pm 1.3 \times 10^{-17}$ at 400 THz). Since we cannot yet verify the source of the offset at 350 THz, we adopt the scatter of the offsets given in Table 1 as the average uncertainty in the frequencies of the comb lines, which is 4×10^{-17} at 400 THz.

In conclusion, we have demonstrated that the stability and accuracy limits of an octave-spanning comb generated with a femtosecond laser are at a sufficiently low level to be useful as a clockwork for the best current optical-frequency standards. As the current results are at the limit imposed by Doppler shifts, active control of all optical paths will be necessary in the future to reach the ultimate stability and accuracy limits.

The authors acknowledge the thoughtful comments and assistance of Th. Udem, J. Bergquist, J. Ye, J. L. Hall, C. Oates, and A. Curtis. We are endebted to R. S. Windeler of Lucent Technologies for providing the microstructure fiber, and are grateful to A. Bartels of GigaOptics GmbH for his assistance with the femtosecond laser.

References

1. Th. Udem, J. Reichert, R. Holzwarth, T. W. Hänsch, Phys. Rev. Lett. **82**, 3568 (1999).

2. Th. Udem, J. Reichert, R. Holzwarth, T. W. Hänsch, Opt. Lett. **24** , 881 (1999).

3. S. A. Diddams, D. J. Jones, J. Ye, S. T. Cundiff, J. L. Hall, J. K. Ranka, R. S. Windeler, R. Holzwarth, Th. Udem, T. W. Hänsch, Phys. Rev. Lett. **84**, 5102 (2000).

4. D. J. Jones, S. A. Diddams, J. K. Ranka, R. S. Windeler, A. J. Stentz, J. L. Hall, S. T. Cundiff, Science **228**, 635 (2000).

5. R. Holzwarth, Th. Udem, T. W. Hänsch, J. C. Knight, W. J. Wadsworth, P. St. J. Russell, Phys. Rev. Lett. **85**, 2264 (2000).

6. Th. Udem, S. A. Diddams, K. R. Vogel, C. W. Oates, E. A. Curtis, W. D. Lee, W. M. Itano, R. E. Drullinger, J. C. Bergquist, L. Hollberg, Phys. Rev. Lett **86**, 4996 (2001).

7. J. Stenger, Ch. Tamm, N. Haverkamp, S. Weyers, H. R. Telle, http://xxx.lanl.gov/abs/physics/0103040.

8. S. A. Diddams, Th. Udem, K. R. Vogel, C. W. Oates, E. A. Curtis, R. S. Windeler, A. Bartels, J. C. Bergquist, L. Hollberg, in Laser Frequency Stabilization, Standards, Measurement, and Applications, J. L. Hall, Jun Ye, eds., Proc. of SPIE Vol. 4269, 77-83.

9. C. W. Oates, E. A. Curtis, L. Hollberg, Opt. Lett. **25**, 1603 (2000).

10. B. C. Young, F. C. Cruz, W. M. Itano, J. C. Bergquist, Phys. Rev. Lett. **82**, 3799 (1999).

11. S. A. Diddams, Th. Udem, J. C. Bergquist, E. A. Curtis, R. E. Drullinger, L. Hollberg, W. M. Itano, W. D. Lee, C. W. Oates, K. R. Vogel, D. J. Wineland, Science (in press).

12. A. Bartels, T. Dekorsy, H. Kurz, Opt. Lett. **24**, 996 (1999).

13. J. K. Ranka, R. S. Windeler, A. J. Stentz, Opt. Lett. **25**, 25 (2000).

14. J. Ye, L.-S. Ma, J. L. Hall, submitted for publication.

15. P. Lesage, IEEE Trans. Inst. & Meas. **32**, 204 (1983).

16. A. A. Madej, J. E. Bernard, in *Frequency Measurement and Control*, A. N. Luiten, ed. (Springer-Verlag, Berlin, 2001), pp. 153-194.

17. R. Holzwarth, Ph.D. dissertation, Ludwig-Maximilians-Universität, München (2001).

List of Figures

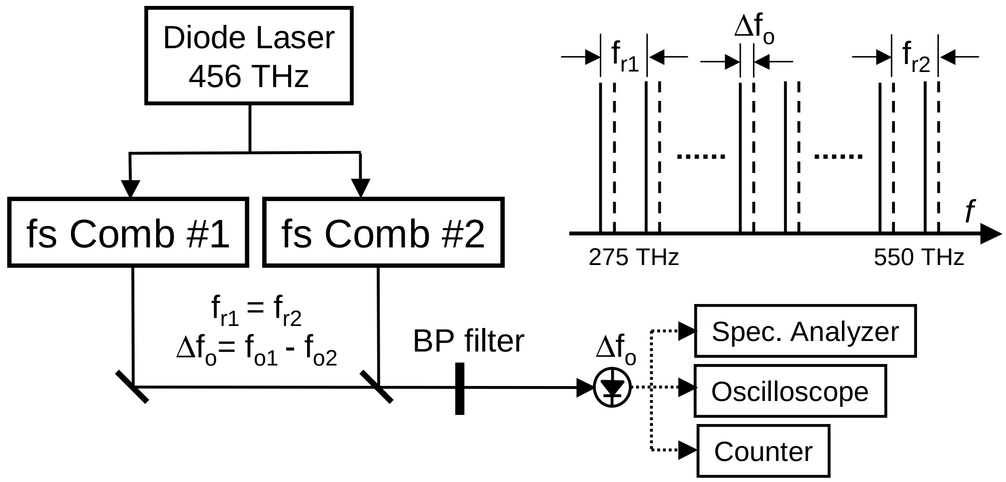

Figure 1, S. A. Diddams et al.

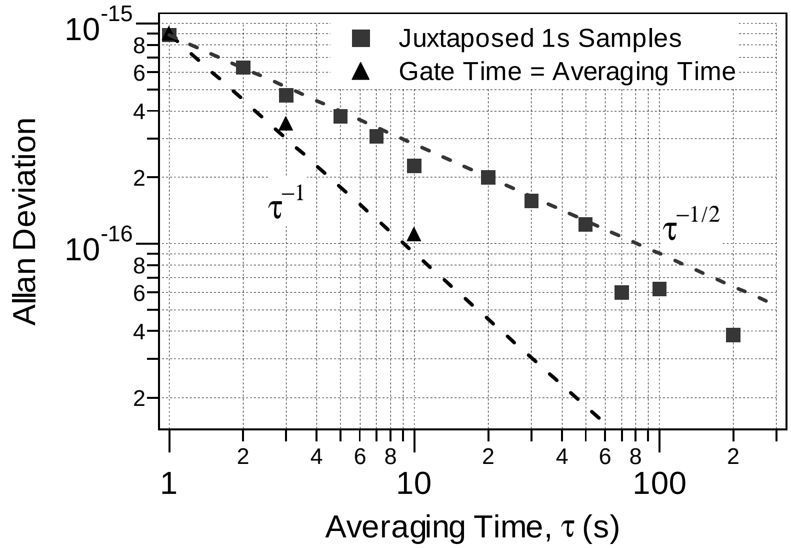

Figure 2, S. A. Diddams et al.

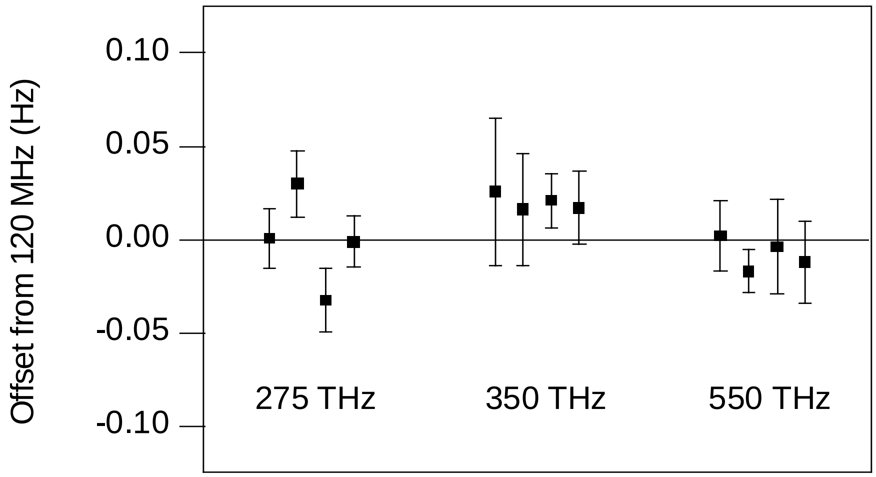

Figure 3, S. A. Diddams et al.

Table 1. Summary of measured stability and offset between the two combs locked to a common laser diode at 456 THz. The fractional offset values at each frequency are the weighted averages of the respective data of Fig. 3.

Freq. (THz)	1 s Allan Dev.	Frac. Offset	Integration Time (s)
275	7.0×10^{-16}	$(-0.5 \pm 2.9) \times 10^{-17}$	539
350	7.2×10^{-16}	$(5.6 \pm 3.0) \times 10^{-17}$	500
550	8.9×10^{-16}	$(-2.0 \pm 1.5) \times 10^{-17}$	1186