FORTY-FIFTH ANNUAL SYMPOSIUM ON FREQUENCY CONTROL

ULTRALINEAR SMALL-ANGLE PHASE MODULATOR

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Abstract

A dc electric field applied to a quartz plate resonator causes changes in the elastic constants which can lead to a change in the frequency of the resonator. This effect, known as the polarizing effect, has been shown to be extremely linear. We have used this effect to build a phase modulator with 2nd-harmonic distortion that is at least 117 dB below the fundamental modulation and low added phase noise. A description of the modulator as well as methods of measurement are discussed.

INTRODUCTION

To find the center of a resonance curve accurately, either phase or frequency modulation techniques are used to guide the servo [1-6]. As the resolution requirements increase so do the requirements on the spectral purity of the modulation reference and the linearity of the modulator. 2nd-harmonic distortion in sine wave modulators can lead to offset errors. The design goals for NIST-7, a new optically pumped cesium beam frequency standard, is that the servo resolve line center to an accuracy of 1 ppm. This requires that the modulation reference and the modulator maintain 2nd-harmonic distortion approximately 114 dB below the fundamental modulation [1,2]. The 2nd-harmonic distortion requirements on the demodulator are approximately -65 dB and not a significant problem [1,2]. Traditional methods of phase modulation have not demonstrated such high linearity; thus a new technique is required. We describe a new type of small-angle analog phase modulator, based on the polarizing effect in quartz resonators, which is exceptionally linear [7-12]. Measurements on three such devices show that the 2nd-harmonic distortion is at least 117 dB below the fundamental modulation. The phase modulator has very low added noise so as to minimize noise contributions at the 2nd-harmonic of the modulation frequency [3]. We also discuss the measurement techniques to verify this performance.

MODULATION ERRORS

If phase modulation of the form

$$\Delta\phi = B \cos \Omega t \tag{1}$$

is used and the error detection is done at the fundamental, 2nd-harmonic distortion with phase cos $2\Omega t$ leads to offset errors [1,2]. Real phase modulators have small nonlinearities which generate small components of modulation at multiples of the modulation frequency.

Assume that the realized phase modulated signal is of the form

$$\omega = \omega_1 + B \cos \Omega t - M_{2s} \sin 2\Omega t + M_{2c} \cos 2\Omega t, \tag{2}$$

where $\Omega/(2\pi)$ is the modulation frequency and $\omega_1/(2\pi)$ is the average frequency of the probe oscillator. Coefficients M_{2s} and M_{2c} contain the effects of second harmonic distortion in the modulation process under the assumption that the residual modulation at harmonics of Ω in the reference signal are small compared to that imposed by the modulator. Neglecting higher harmonic effects and nonlinearities in the detection process, the offset in the error signal is [2]

$$\text{Frequency offset} = -1/2\ M_{2c}. \tag{3}$$

The NIST-7 servo requires resolving line center to an accuracy of 1 part in a million. Therefore the 2nd-harmonic distortion of the phase represented by M_{2c} must be less than 2×10^{-6}B or at least 114 dB below the fundamental modulation [1,2].

POLARIZING EFFECT MODULATOR-THEORY

When a vibrating piezoelectric resonator is subjected to a dc electric field, ΔE, it responds by a change, $\Delta\nu$ in its resonance frequency ν [7-13]. The polarizing effect in quartz resonators is so nearly linear that it can be adequately described using only the 1st- and 2nd-order terms of the Taylor expansion about $E = 0$. Therefore,

$$\Delta\nu/\nu = P_L\ \Delta E + P_Q\ \Delta E^2, \tag{4}$$

where

$$P_L = 1/\nu(d\nu/dE) \text{ at } E=0 \text{ and} \tag{4A}$$

$$P_Q = 1/(2\nu)\ (d^2\nu/dE^2) \text{ at } E=0. \tag{4B}$$

For a resonator to exhibit a polarizing effect there must be a component of E along the x-axis of the quartz resonator. Additional discussion including the effects of ion migration altering the sensitivity for dc applied electric fields are found in [11-13]. Typical values for P_L and P_Q are on the order of 10^{-11} to 10^{-12} and 10^{-20} to 10^{-21}, respectively, depending on the crystal cut and overtone used [7-9].

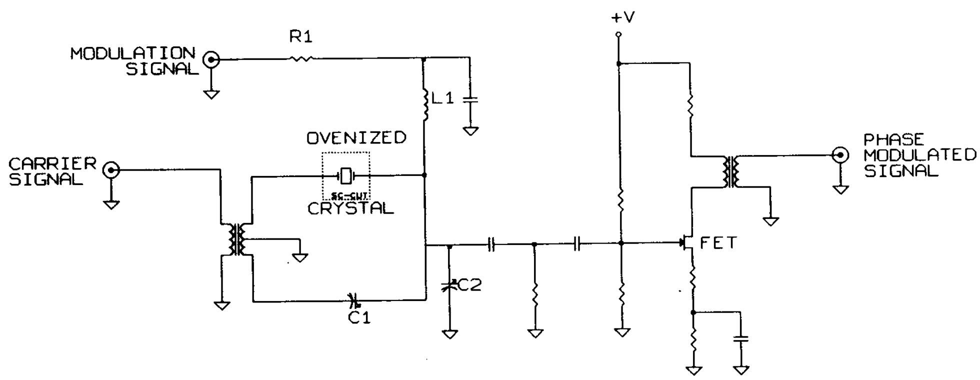

Figure 1. Block diagram of the new phase modulator.

Near line center the steady state phase shift across a high Q resonator is

$$\Delta\phi = 2Q\,(\nu - \nu_0)/\nu_0, \tag{5}$$

where ν is the applied frequency, ν_0 is the resonance frequency of the resonator, and Q is the loaded quality factor of the resonator. Thus a frequency change $\Delta\nu$ in the frequency of the resonator generated by applying an electric field E across the resonator results in a phase shift

$$\Delta\phi = 2Q(P_L\,\Delta E + P_Q\,\Delta E^2). \tag{6}$$

POLARIZING EFFECT MODULATOR-PRACTICAL REALIZATION

Figure 1 shows a generalized schematic of the polarizing phase modulator. A balanced transformer and adjustable capacitor C_1 are used to minimize off resonance transmission. A 3rd overtone SC-cut 5 MHz resonator was chosen because the polarizing coefficient was much larger [8] and both the static and dynamic temperature coefficients are much better than for comparable AT-cut units [13]. The resonator was temperature controlled in a standard crystal oven at 80°C and tuned to resonance by adjusting C_2. Tuning to resonance was determined by nulling the amplitude modulation observed on a detector diode placed at the output when a slow modulation voltage is applied across the resonator through R_1 and L_1. No other alignment or adjustment is necessary. The isolation amplifier on the output of the modulator provides overall unity gain and removes the effects of changing loads on the tuning of the resonator.

For NIST-7, a phase modulation of approximately 0.55 mrad is required at 5 MHz to achieve a phase modulation of 1 radian at the final output frequency of 9.2 GHz. The desired modulation frequency $\Omega/(2\pi)$ = 49 Hz is much larger than the half-bandwidth of the resonator. At such high modulation frequencies the phase shift across the resonator becomes

$$\Delta\phi = [2Q(P_L\Delta E + P_Q\Delta E^2)][1 + ((Q\Omega/(\pi\nu_0))^2]^{-1}, \tag{7}$$

where $\nu_0/(2Q)$ is the loaded half-bandwidth of the resonator.

Figure 2 shows the relative amplitude of the phase modulation as a function of modulation frequency. These results follow Eq. (7) within $\pm$ 0.5 dB for modulation frequencies from 0.1 up to at least 400 Hz.

Approximately 7 V (6400 V/m) RMS is required to achieve the desired modulation level using a, 3rd overtone SC-cut 5 MHz quartz crystal and a modulation frequency of 49 Hz. This corresponds to a dc linear polarizing sensitivity of P_L = 2.6 $\pm$0.3 x 10^{-12} m/V. The difference between our data (which is about a factor of 5 smaller) and that found by Hruska [7,8] is probably due to the difference in the electrode patterns, cut angle, and the shape. The measurements of P_L for a particular resonator can easily be made to an accuracy of a few percent with our technique, which is free from effects due to ion migration [11-13]. From the relative values of P_L and P_Q from [8] and the shape of Fig. 3 we would expect that the second harmonic distortion due to the resonator to be approximately 126 dB below the fundamental. Even lower harmonic distortion would be expected if the modulation frequency was lower since a lower voltage would be required to achieve the same phase modulation angle.

The added phase noise is somewhat lower (typically $\mathcal{L}(10)$ = -134 dBc/Hz and $\mathcal{L}(100)$ = - 154 dBc/Hz) than the present 5 MHz oscillator. (Techniques for measuring the added phase noise of components are discussed in [14,15].) Further improvements in the phase noise appear possible. The added white frequency contribution of a passive standard due to the present level of added phase noise at the second harmonic of the modulation frequency (2Ω = 100 Hz) is approximately 2 x 10^{-13} $\tau^{-1/2}$ [3].

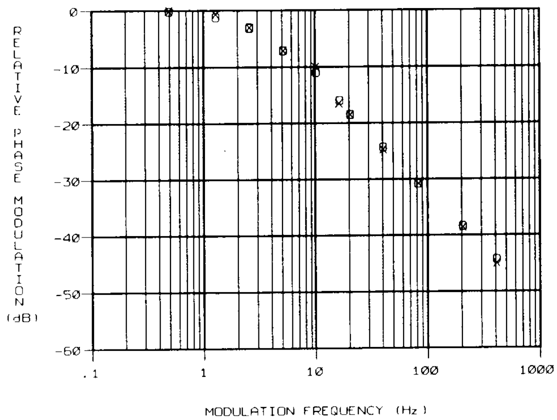

Figure 2. Relative phase modulation of the output signal as a function of the frequency of modulation. The open circles are the predictions using Eq. (7) and the half-bandwidth $\nu_0/(2Q)$ measured from transmission experiments. The solid points are the phase modulation levels obtained from phase bridge measurements [14,15].

MEASUREMENTS OF HARMONIC DISTORTION

The dynamic range of most spectral analyzers--from 60 to 100 dB--is not enough to observe 2nd-harmonic levels expected to be 120 dB below the fundamental. Some filtering is therefore required to extend the dynamic range to this level. The general approach is to arrange the various tests so that the fundamental modulation signal is nulled against another signal by at least 40 dB before interacting with a phase detector, spectrum analyzer, or other nonlinear device. We compared 3 different devices to determine the distortion characteristics of each modulator. Figure 3 shows the test setup used to compare the difference in phase modulation of two modulators. The mixer is sensitive to only the difference in the phase modulation level between the two modulators. The input reference level to one modulator is adjusted to achieve the required 0.55 mrad phase modulation. The input modulation signal to the other modulator is adjusted in amplitude and phase to null the demodulated fundamental signal at the output of the mixer. Typical reduction of the fundamental was 40 to 60 dB.

The out-of-phase distortion products between two modulators at frequency Ω/π were measured by driving the second modulator with the output signal from the first modulator, applying modulation signals that are 180° out of phase and using the unmodulated signal from the power splitter to drive the reference side of the mixer. See Fig. 4. Great care must be taken not to introduce spurious signals due to residual amplitude modulation since typical double balance mixers only provide 30 to 40 dB of AM suppression. The worst case 2nd-harmonic distortion between the three modulators was -117 dB relative to the fundamental modulation. A typical result is shown in Fig. 5. The fundamental modulation corresponds to -6 dBV, yielding anet sum for the second harmonic distortions of -126 dB for this pair.

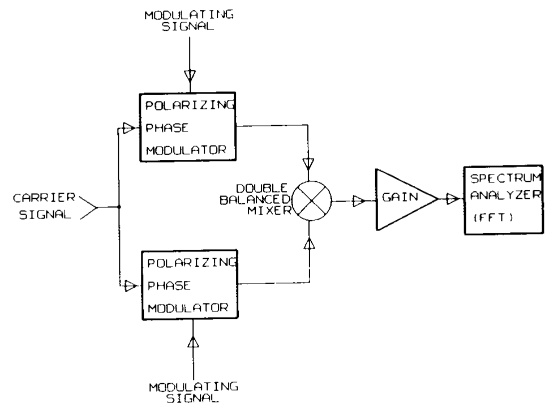

Figure 3. Block diagram of a parallel phase bridge measurements to determine the difference in phase modulation between two phase modulators [14,15].

In this configuration the output of the mixer is of the form

$$\phi_{mod} = V_0\cos\omega t + (B_1\text{-}B_2)\cos\Omega t + (M^1_{2C} + M^2_{2C})\cos 2\Omega t + (M^1_{2S} + M^2_{2S})\sin 2\Omega t \qquad (8)$$

under the assumption that all of the 2nd harmonic distortion is due to the modulation process.

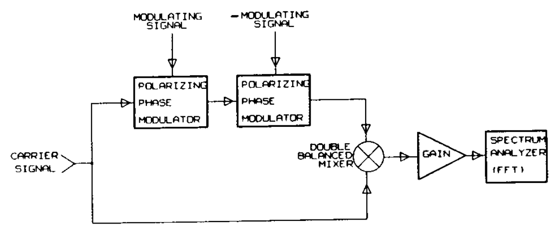

Figure 4. Block diagram of a series phase bridge measurements to determine the sum of the 2nd harmonic in phase modulation between two phase modulators [14,15].

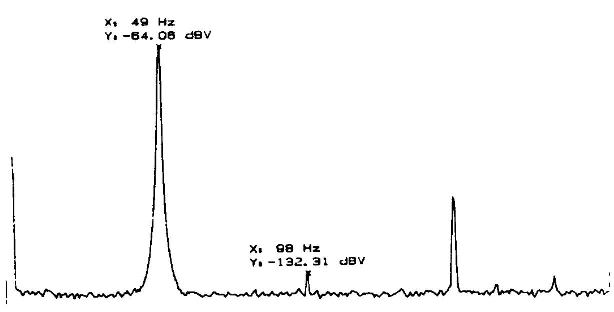

Figure 5. Typical result of a series phase bridge measurement.

CONCLUSIONS

We have introduced a method for measuring the polarizing effect in high-Q SC-cut 5 MHz quartz resonators for small ac electric fields and found that the sensitivity and the linearity is in general agreement with the previous work at dc [7-11]. We have demonstrated that this polarizing effect can be used to make an ultralinear small angle phase modulator with low added phase noise. The second harmonic distortion was measured to be less than -117 dB relative to the fundamental in three different units even though the modulation frequency was more than 20 times larger than the half-bandwidth of the resonators. Using low modulation frequencies, the second harmonic distortion could probably approach -140 dB relative to the fundamental modulation. The high linearity, low noise, and simplicity of the circuit combine to make this new small angle analog phase modulator an interesting tool in precision frequency metrology.

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