Environmental Sensitivities of Quartz Crystal Oscillators

Fred L. Walls
Time and Frequency Division
325 Broadway
Boulder, Colorado 80303

Abstract

The frequency, amplitude, and noise of the output signal of a quartz-crystal-controlled-oscillator is affected by a large number of environmental effects. This paper examines the physical basis for the sensitivity of precision oscillators to temperature, humidity, pressure, vibration, magnetic field, electric field, load, and radiation. The sensitivity of crystal oscillators to radiation is a very complex topic and poorly understood. Therefore only a few general results are mentioned. The sensitivity to most external influences often varies significantly from one oscillator type to another and from one unit of a given type to another. For a given unit, the sensitivity to one parameter often depends on the value of other parameters and history. Representative sensitivity to the above parameter will be given.

I. Introduction

Quartz crystal oscillators are a fundamental element in many areas of frequency metrology affecting applications such as communication and navigation. Their frequency, output level, amplitude noise, and phase noise are generally critical parameters that determine the overall performance of a system. In many applications their performance is significantly less than that obtained under ideal environmental conditions. In this paper I will briefly outline the physical basis and representative values for the sensitivity of quartz crystal oscillators to environmental parameters. The most important of these are temperature, humidity, pressure, vibration, magnetic field, electric field, load, and radiation. The sensitivity of quartz oscillators to radiation is a very complex and poorly understood topic and therefore only a few general results are mentioned. For a given oscillator the sensitivity to one parameter often depends on the value of other parameters, and the history of the device. It is often difficult to separate the influence of one parameter from that of another. Several methods for characterizing the environmental sensitivities of both quartz resonators and oscillators are discussed in [1]. Much more effort is needed in this area.

II. Model of a Quartz Controlled Oscillator

Figure 1 shows a simplified diagram of a quartz-crystal-controlled oscillator. The well known oscillation conditions are that the phase around the loop be an integral multiple of $2\pi$ and that the loop gain be 1 [2-5]. Small changes in the loop phase, $d\phi$, are compensated by a change in oscillation frequency of
$\Delta \nu/\nu_0 = (1/2Q) \Delta \omega.$

where $\nu_0$ is the oscillation frequency and $Q$ is the loaded Q-factor of the resonator. Small changes in the frequency of the resonator are directly translated into changes in the frequency of the oscillator since the rest of the electronics is generally very broadband compared to the resonator, that is.

$$\Delta \nu/\nu_{0,\text{oscillator}} = d\nu/\nu_{0,\text{resonator}}. \quad (2)$$

These two equations provide the basis for understanding how the various environmental conditions affect both the short-term and the long-term frequency stability of the oscillators.

III. Changes Within the Resonator

Environmental effects that change the frequency of the resonator have been investigated by many people in much more detail than can be described in this paper. The most important environmentally driven changes within the resonator are driven by changes in temperature, level of excitation (rf amplitude), stress, adsorption and desorption of material on the surface, vibration, radiation, electric field, and magnetic field.

The frequency shifts due to these effects are universal to the extent that a given change in resonator frequency causes the same change in oscillator frequency in the limit that the oscillator loop is broadband. Different oscillator designs have an influence on the apparent isolation of the resonator from the environmental parameters.

A. Temperature and Temperature Changes

Temperature variations change the value of the elastic constants and, to a lesser degree, the dimensions of the resonator. The change in resonator frequency with temperature varies greatly with crystallographic cut and orientation [6-21]. Figure 2 shows a typical static frequency versus temperature curve for precision quartz resonators [16]. The actual values depend on the resonator cut, overtone, frequency, diameter, and mounting technique. Temperature changes and temperature gradients often cause frequency changes that are large compared to the slope of the static curve [6-22]. Typical coefficients for the frequency-temperature effect for 5 MHz resonators are

$$\Delta \nu/\nu_0 = 10^{-9} \Delta T^2 - 10^{-5} dT/dt \quad (3)$$

for 5th overtone AT-cuts, and

$$\Delta \nu/\nu_0 = 10^{-9} \Delta T^2 + 10^{-7} dT/dt \quad (4)$$

for 3rd overtone SC-cuts. Here $\Delta T$ is the temperature difference in K from turnover and $dT/dt$ is the rate of change of temperature in K/s.
The dynamic temperature effect leads to hysteresis in the experimental measurements of temperature coefficients as shown in Figs. 3 and 4. This effect makes it difficult to locate the exact turn-over point. The in turn leads to oscillators with finite frequency changes even for slow, small temperature excursions, especially for AT-cut resonators. Table 1 shows typical temperature coefficients for an AT-cut resonator as a function of the error in setting the oven to the exact turn-over point. Table 2 shows typical temperature coefficients for an SC-cut resonator as a function of the error in setting the oven to the exact turn-over point. The actual change in temperature may be driven by changes in atmospheric pressure and/or humidity thereby changing the thermal conductance and temperature gradients within the oscillator package [22-24]. Deposited radiative energy can also change the temperature of the resonator. The performance potential of AT-cut resonators is extremely difficult to attain due to their very high dynamic temperature coefficient. Sulzer was the first to attain a performance $\sim 3 \times 10^{-13}$ from AT-cut resonators, and it was many years before it was generally realized that the reason for the excellent performance was an oven design with extremely low thermal transients [25]. The reduction in the dynamic temperature coefficient for SC-cut resonators as compared to the earlier AT- and BT-cut resonators represents a major advance in the practical application of quartz crystal oscillators in nonideal environments using only simple ovens. Much better thermal performance can be obtained using multiple ovens [26], aged high-performance thermistors [27], and compensated oven designs [28]. The tradeoff is increased complexity, size, weight, and cost.

Temperature generally does not affect the phase noise or short-term frequency stability of an oscillator. However, in cases where the sustaining electronics is not temperature controlled, slight changes in gain and noise figure with temperature will be reflected in the output phase noise. Reviews of the correlation of output phase noise with the noise performance of the sustaining stage and output amplifier are given in [2-5, 29].

Another important effect, especially with AT-cut resonators, is activity dips. Activity dips are due to the accidental overlap of some other mode of the resonator with the resonance mode. This coupling to the unwanted mode leads to increased losses and hence a reduction in the oscillator amplitude. The frequency of the resonator is also pulled by the coupling to the other mode, which usually has much higher sensitivity to temperature than the primary mode of oscillation. This leads to temperature-frequency coefficients that vary rapidly with temperature over very narrow ranges in temperature of the resonator [30-34]. Figure 5 shows the frequency-versus-temperature performance of the primary clock in the Ginga satellite [33]. The nonlinear thermal coefficient near $18^\circ C$ make it very difficult to model to accurately recover clock timing. SC-cut resonators and some types of lateral field resonators show much reduced incidence of activity dips and their associated quirks in the temperature coefficients [34-36].

The dynamic temperature effect and the possibility of an activity dip significantly complicates the specification and measurement of oscillator temperature coefficients. For critical applications the frequency must be measured over the entire operating temperature range using a model of the actual temperature profiles.

B. RF Excitation Level

The frequency of the resonator is also a function of amplitude of the signal level as shown in Fig. 6 [16, 18-20, 34, 37-39]. The sensitivity to this effect, usually called the amplitude-frequency effect, is a function of the blank curvature as shown in Fig. 7 [34]. Typical sensitivities to this parameter range from approximately $10^{-9}/\mu W$ for 5th overtone AT- or BT-cut resonators to parts in $10^{-11}/\mu W$.
for 3rd overtone. SC-cut resonators at 5 MHz. The primary environmental drivers for this effect are temperature, humidity, or radiation changing the excitation level through interaction with the automatic gain control (AGC) and the gain of the sustaining stage. Changes in the dc supply voltage can also affect the amplitude of oscillation by changing the AGC circuitry.

C. Stress (Force)

Stress on the resonator blank changes the resonance frequency through the nonlinear piezoelectric coefficients [11-21, 34, 40-44]. Stress is transmitted to the resonator through the mounting structure. It can originate from temperature-driven dimensional variations in the vacuum enclosure, changes in the pressure surrounding the vacuum enclosure, changes in the magnetic field causing a change in the mounting force due to the use of magnetic components. The stress on the resonator, due to the use of electrodes directly plated onto the resonator, can change with resonator drive, temperature, radiation exposure, and time. The stress due to the electrode has been estimated [12-13, 20], but the changes with environmental effects such as vibration and temperature cycling are very difficult to estimate. Figure 8 shows the change in frequency of a traditional AT-cut plate due to diametrically opposed forces in the plane of the resonator as a function of the angle between the applied force and the x axis [29]. BVA resonators are probably less sensitive to this effect due to their unique mounting arrangement [20-21].

D. Adsorption-Desorption

Changes in the quantity or distribution of molecules on the surface of the resonator can lead to very large changes in the frequency of the resonator [6, 16, 46-48]. One monolayer added to a 5 MHz resonator amounts to roughly 1 part per million change in the frequency [16]. Major drivers of changes in the background pressure and in the movement of adsorbed gasses are temperature changes and enclosure outgassing or leaks.

The background pressure of helium inside the resonator enclosure can significantly increase if the vacuum enclosure is glass and the resonator is operated in an environment with large amounts of helium. A typical helium leak rate for a glass enclosure operating at 80°C in a pure helium environment is $5 \times 10^{-3} \text{Pa/s}$. Even in air at 80°C the helium builds up at a rate of approximately $2 \times 10^{-8} \text{Pa/s}$ or about $0.7 \text{Pa/yr}$ [49]. The use of metal enclosures greatly reduces the helium leak rate, but most metals outgas significant amounts of hydrogen and may contribute to the drift of some oscillators [29,49]. A typical sensitivity for an AT-cut resonator to a non-reactive gas is $10^{-7} / \text{Pa}$ (0.7 $\times 10^{-10} / \text{Torr}$). Ceramic enclosures that reduce the helium and hydrogen leak rates to negligible values have been developed [46]. There also is some question as to what portion of the residual phase noise in quartz resonators is due to time varying rates of collisions with the background gas [48].

E. Acceleration/Vibration

Although a large shock and/or vibration can change the long-term frequency of the resonator, the dominant effect is usually the instantaneous change in the frequency of the resonator due to changes in the stress applied to the resonator through the mounting structure [6,14,16,18-21,50-57]. The frequency change depends on orientation and is linear with applied acceleration or vibration up to approximately 50 $g$ [19]. See Fig. 9. The maximum sensitivity is typically of order $2 \times 10^{-9} / g$. Significant effort has
been expended in minimizing this effect through compensation [32-34], mounting techniques [20-21, 19-30, 32], and resonator fabrication techniques [20-21]. The net sensitivity for specially compensated or fabricated oscillators ranges from approximately $10^{-11}$ to $3 \times 10^{-10}$ per g. The change in frequency due to inversion "2g tipover test" is often biased due to changes in the temperature gradient as shown in Fig. 10. Although not commonly mentioned, magnetic field sensitivity can also bias the test since the magnetic field effect also is a function of position and motion [58-59].

The phase noise of a resonator subjected to vibration is increased by an amount

$$L(f) = \frac{1}{4} \left( \frac{\Gamma^2}{f^2} \right) A^2,$$

where $\Gamma$ is acceleration sensitivity and $A$ is the applied acceleration. Figure 11 shows the quiescent phase noise of a 100 MHz oscillator and that obtained with $\Gamma = 2 \times 10^{-9}/g$ and $A = 2 \cos 2\pi f t$. The increase in phase noise over that obtained under quiescent conditions is approximately 70 dB at a Fourier frequency of 10 kHz. Even with $\Gamma = 1 \times 10^{-11}/g$ the degradation would be about 30 dB.

F. Radiation

Radiation interacts with the resonator in many ways. Although not fully characterized for each resonator type and oscillator, it is possible to list some common aspects. A more detailed summary is found in [16].

Pulse Irradiation Results

1. For applications requiring circuits hardened to pulse irradiation, quartz resonators are the least tolerant element in properly designed oscillator circuits.

2. Resonators made of unswept quartz or natural quartz can experience a large increase in series resistance, $R_s$, following a pulse of radiation; the radiation pulse can even stop the oscillation.

3. Resonators made of properly swept quartz experience a negligible change in $R_s$ when subjected to pulsed ionizing radiation (the oscillator circuit does not require a large reserve of gain margin).

Steady-State Radiation Results

1. At doses <100 rad, frequency change is not well understood. Radiation can induce stress relief. Surface effects such as adsorption, desorption, dissociation, polymerization and charging may be significant. The frequency change is nonlinear with dose.

2. At doses >1 Krad, frequency change is quartz impurity dependent. The ionizing radiation produces electron-hole pairs; the holes are trapped by the impurity Al sites while the compensating cation (e.g. Li or Na) is released. The freed cations are loosely trapped along the optic axis. The lattice near the Al is altered, and the elastic constant is changed; therefore, the frequency shifts. Ge impurities are also troublesome.

3. At 10^6 rad dose, frequency change ranges from $10^{-11}$ per rad for natural quartz to $10^{-14}$ per rad for high quality swept quartz.
4. Frequency change is negative for natural quartz; it can be positive or negative for cultured and swept cultured quartz.

5. Frequency change saturates at doses $> 10^6$ rads.

6. The $Q$ degrades if the quartz contains a high concentration of alkali impurities; the $Q$ of resonators made of properly swept cultured quartz is unaffected.

7. Frequency change anneals at $T > 240^\circ C$ in less than 3 hours.

8. Preconditioning (e.g., with doses $> 10^5$ rads) reduces the high dose radiation sensitivities upon subsequent irradiations.

9. High dose radiation can also rotate frequency-vs.-temperature characteristic.

G. Electric Field

The frequency of certain resonator cuts are directly affected by the application of even small electric fields through changes in dimension and effective mass and through interaction with the nonlinear coefficients [58-63]. The application of electric fields also tends to cause ions within the crystal to move which changes the frequency. The net result is that the change in frequency generally has a fast component due to the interaction with the crystal constants and one or more slower components associated with the movement of ions as shown in Fig. 12. The slower time constants depend exponentially on temperature. This electric field effect has been used to vibration compensate SC cut resonators [54] and to create an ultra-linear phase modulator [60]. Sensitivities to this effect are highly cut and material dependent and range from approximately $10^{-11}$ to $10^{-8}$ per volt applied across the resonator. Large electric fields and elevated temperatures are sometimes used to “sweep” ions out of the quartz bar prior to fabrication resonators [16,60-63]. This is most often used on resonators for radiation environments.

H. Magnetic Field

The inherent magnetic field sensitivity of quartz resonators is probably smaller than $10^{-11}/T$ [1,64-66]. Most resonators are, however, constructed with magnetic holders. As the magnetic field changes the force on the various components of the resonator. This causes a frequency shift through the force-frequency coefficient discussed above.

IV. Changes Within the Loop Electronics

Many environmental parameters cause change in the phase around the oscillator loop. The most important are temperature, humidity, pressure, acceleration/vibration, magnetic field, voltage, load, and radiation. This environmental sensitivity often leads to increases in the level of wide-band phase noise in the short-term, random-walk frequency modulation in the medium-term, and drift in the long-term. The value of loop phase shift are not universal, but critically depend on the circuit design and the loaded $Q$-factor of the oscillator.
A. Tuning Capacitor

The frequency of the oscillator is generally fine-tuned using a load capacitor, $C_L$, of order 20 to 32 pF. In practice this capacitor is often made up of a fixed value (selected at the time of manufacture), a mechanically tuned capacitor for coarse tuning; a varactor for electronic tuning, and a contribution from the input capacitance of the sustaining stage and matching networks. The frequency change for small changes in $C_L$ are

$$\Delta \nu / \nu = \frac{C_1/2}{C_0 + C_L} \frac{dC_L}{C_0 + C_L},$$

where $C_0$ is the parallel capacitance and $C_1$ is the motional capacitance. It is not uncommon for the first term in Eq. 5 to be of order $10^{-3}$ [16]. In this case a change in $C_L$ of only $10^{-6}$ results in a frequency change of $10^{-9}$. Primary environmental parameters which change $C_L$ are temperature, humidity, pressure, and shock/vibration. Humidity and pressure change the ratio between convection and conduction cooling. See Figs. 10 and 13. Pressure changes in a hermetically sealed oscillator can also be driven by temperature. This changes the temperature gradients and thereby the temperature of the tuning capacitor (and also the temperature of the resonator.) Shock and vibration can change the mechanical capacitor, if present. Changes in the supply voltage, AGC and temperature all change the input capacitance of the sustaining stage and the varactor diode. Humidity can change the value of the dielectric coefficient and losses in the capacitors and even the circuit board. Radiation can change the gain and offsets of the AGC and the sustaining stage and thereby change the effective input capacitance.

B. Mode Selection/Tuned Circuits

Most oscillators use matching circuits and filters to adjust the loop phase to approximately $0\pi$ and, especially with SC-cut resonators, to suppress unwanted modes. The phase shift across a transmission filter is given approximately by Eq. 1 with the $Q$ in this case being that of the tuned circuit, $Q_c$. The fractional change in output frequency due to small changes in either circuit inductance, $L_c$, or circuit capacitance, $C_c$, is approximately given by

$$\Delta \nu / \nu \sim \frac{1}{2Q} \sim \frac{Q_c}{Q} \frac{dc}{C_c} + \frac{dL}{L_c}.$$  

Far from resonance the change in loop phase with change in filter capacitance or inductance is generally much less than that given by Eq. (1). This suggests that the use of notch filters to suppress unwanted modes is probably superior to the use of narrow-band transmission filters.

The values of most resistors, inductors and capacitors, and even parameters associated with the active junctions, are a function of temperature, humidity, current, or voltage. Significant improvements in the medium-term frequency stability can often be obtained merely by sealing an oscillator to prevent changes in the humidity and pressure [22-23]. Figure 13 shows the frequency change of a high performance oscillator due to a change in relative humidity from approximately 20% to 100% [22-23]. This fractional change of frequency is about 1000 times the normal 1 s frequency stability of $3 \times 10^{-13}$.  

471
C. External Load

If the external load of the oscillator changes, there is a change in the amplitude and/or phase of the signal reflected back into the oscillator. The portion of this reflected signal that reaches the oscillating loop changes the phase of the oscillation and hence the output frequency by an amount given by Eq. (1) and reexpressed in Eq. (8). In this case we can estimate the maximum phase change as just the square root of the isolation. For example 40 dB of isolation corresponds to a maximum phase deviation of $10^{-3}$ radians.

$$\Delta \nu / \nu \sim \frac{1}{2Q} d \phi \sim \frac{q}{2Q} \sqrt{\text{isolation}}$$

For $Q \sim 10^6$ and isolation of 40 dB, the maximum pulling is approximately $\Delta \nu / \nu = 5 \times 10^{-9}$. This is an approximate model for most 5 MHz oscillators. As the frequency increases the problem of load pulling becomes much worse because both the Q-factor and the isolation decrease.

D. Acceleration, Vibration, and Magnetic Fields

Acceleration and vibration can distort the circuit substrate and the position of components leading to changes in the stray inductance and/or capacitance and thereby changes in the frequency. Virtually none of the presently available oscillators have magnetic shielding. The presence of magnetic field complicates matters significantly since changes in orientation within the magnetic field lead to frequency shifts and motion dependent effects. Therefore movement, acceleration, and/or vibration certainly leads to induced electric and magnetic fields that disturb the quiescent performance of the oscillator. In most cases such effects are very difficult to separate from other motion induced effects. Acceleration and vibration of circuit elements in the absence of a magnetic field lead to phase/frequency modulation. These effects are difficult to separate from those due to changes of stress applied to the resonator, but are no doubt present and may in some cases prevent accurate measurements on some low-g-sensitive resonators [1,56,57,64-66].

Discussion

The frequency stability of quartz-crystal-controlled oscillators has been refined to the point that small changes in a wide variety of environmental parameters are now significant. The most important environmental parameter is probably acceleration and vibration at frequencies less than approximately 100 Hz. All of the other environmental drivers can be significantly reduced by appropriate attention to circuit design and/or shielding. Among the remaining environmental drivers, temperature is probably the most significant. Most oscillator ovens could be significantly improved using better circuits and better insulation. Magnetic shielding may be necessary to actually realize the full performance of low-g sensitivity resonators. Pressure and humidity effects can be very serious in open oscillators. Fortunately these effects can be eliminated by hermetically sealing the oscillator. The sensitivity to low doses of radiation is not well understood. More work needs to be done to refine the characterization of sensitivity to various environmental parameters.
Acknowledgements

I am particularly grateful to A. Ballato, R. Besson, R. Filler, J. J. Gagnepain, and J. Vig for many discussions of systematic effects in quartz crystal-controlled oscillators.

References


* Copies available from National Technical Information Service, Sills Bldg., 5285 Port Royal Road, Springfield, VA 22116.

† Copies available from Institute of Electrical & Electronics Engineers, 445 Hoes Lane, Piscataway, NJ 08854.
Table 1. Typical frequency versus temperature coefficients for an AT-cut resonator with a turnover temperature of $85^\circ C$ as a function of oven parameters. Frequency stability of $1 \times 10^{-13}$ requires $\Delta T/dt < 10nK/s$.

$$\Delta \nu/\nu \approx 3 \times 10^{-8} \Delta T^2 + 10^{-5} \Delta T/dt$$

<table>
<thead>
<tr>
<th>Oven Offset (mK)</th>
<th>Oven Change (mK)</th>
<th>$\pm 10$ mK</th>
<th>$\pm 1$ mK</th>
<th>$\pm 0.1$ mK</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$3 \times 10^{-10}$</td>
<td>$3 \times 10^{-12}$</td>
<td>$3 \times 10^{-14}$</td>
<td>$3 \times 10^{-15}$</td>
</tr>
<tr>
<td>1</td>
<td>$3 \times 10^{-10}$</td>
<td>$5 \times 10^{-12}$</td>
<td>$2 \times 10^{-13}$</td>
<td>$6 \times 10^{-15}$</td>
</tr>
<tr>
<td>10</td>
<td>$5 \times 10^{-10}$</td>
<td>$2 \times 10^{-11}$</td>
<td>$6 \times 10^{-13}$</td>
<td>$6 \times 10^{-14}$</td>
</tr>
<tr>
<td>100</td>
<td>$2 \times 10^{-9}$</td>
<td>$6 \times 10^{-11}$</td>
<td>$6 \times 10^{-12}$</td>
<td>$6 \times 10^{-13}$</td>
</tr>
</tbody>
</table>

Table 2. Typical frequency versus temperature coefficients for a SC-cut resonator with a turnover temperature of $85^\circ C$ as a function of oven parameters. Frequency stability $1 \times 10^{-13}$ requires $\Delta T/dt < 330nK/s$.

$$\Delta \nu/\nu = 4 \times 10^{-9} T^2 + 3 \times 10^{-7} \Delta T/dt$$

<table>
<thead>
<tr>
<th>Oven Offset (mK)</th>
<th>Oven Change (mK)</th>
<th>$\pm 10$ mK</th>
<th>$\pm 1$ mK</th>
<th>$\pm 0.1$ mK</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4 \times 10^{-11}$</td>
<td>$4 \times 10^{-13}$</td>
<td>$2 \times 10^{-15}$</td>
<td>$4 \times 10^{-17}$</td>
</tr>
<tr>
<td>1</td>
<td>$4 \times 10^{-11}$</td>
<td>$6 \times 10^{-13}$</td>
<td>$2 \times 10^{-14}$</td>
<td>$8 \times 10^{-16}$</td>
</tr>
<tr>
<td>10</td>
<td>$6 \times 10^{-11}$</td>
<td>$2 \times 10^{-12}$</td>
<td>$8 \times 10^{-14}$</td>
<td>$8 \times 10^{-15}$</td>
</tr>
<tr>
<td>100</td>
<td>$2 \times 10^{-10}$</td>
<td>$8 \times 10^{-12}$</td>
<td>$8 \times 10^{-13}$</td>
<td>$8 \times 10^{-14}$</td>
</tr>
</tbody>
</table>
Table 1. Typical frequency versus temperature coefficients for an AT-cut resonator with a turnover temperature of 85°C as a function of oven parameters. Frequency stability of $1 \times 10^{-13}$ requires $\Delta T/\Delta t < 10 \text{nK/s}$.

$$\Delta \nu/\nu = 3 \times 10^{-8} \Delta T^2 + 10^{-5} \Delta T/\Delta t$$

<table>
<thead>
<tr>
<th>Oven Offset (mK)</th>
<th>±100 mK</th>
<th>±10 mK</th>
<th>±1 mK</th>
<th>±0.1 mK</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$3 \times 10^{-10}$</td>
<td>$3 \times 10^{-12}$</td>
<td>$3 \times 10^{-14}$</td>
<td>$3 \times 10^{-15}$</td>
</tr>
<tr>
<td>1</td>
<td>$3 \times 10^{-10}$</td>
<td>$5 \times 10^{-12}$</td>
<td>$2 \times 10^{-13}$</td>
<td>$6 \times 10^{-15}$</td>
</tr>
<tr>
<td>10</td>
<td>$5 \times 10^{-10}$</td>
<td>$2 \times 10^{-11}$</td>
<td>$6 \times 10^{-13}$</td>
<td>$6 \times 10^{-14}$</td>
</tr>
<tr>
<td>100</td>
<td>$2 \times 10^{-9}$</td>
<td>$6 \times 10^{-11}$</td>
<td>$6 \times 10^{-12}$</td>
<td>$6 \times 10^{-13}$</td>
</tr>
</tbody>
</table>

Table 2. Typical frequency versus temperature coefficients for a SC-cut resonator with a turnover temperature of 85°C as a function of oven parameters. Frequency stability $1 \times 10^{-13}$ requires $\Delta T/\Delta t < 330 \text{ nK/s}$.

$$\Delta \nu/\nu = 4 \times 10^{-9} \Delta T^2 + 3 \times 10^{-7} \Delta T/\Delta t$$

<table>
<thead>
<tr>
<th>Oven Offset (mK)</th>
<th>±100 mK</th>
<th>±10 mK</th>
<th>±1 mK</th>
<th>±0.1 mK</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$4 \times 10^{-11}$</td>
<td>$4 \times 10^{-13}$</td>
<td>$2 \times 10^{-15}$</td>
<td>$4 \times 10^{-17}$</td>
</tr>
<tr>
<td>1</td>
<td>$4 \times 10^{-11}$</td>
<td>$6 \times 10^{-13}$</td>
<td>$2 \times 10^{-14}$</td>
<td>$8 \times 10^{-16}$</td>
</tr>
<tr>
<td>10</td>
<td>$6 \times 10^{-11}$</td>
<td>$2 \times 10^{-12}$</td>
<td>$8 \times 10^{-14}$</td>
<td>$8 \times 10^{-15}$</td>
</tr>
<tr>
<td>100</td>
<td>$2 \times 10^{-10}$</td>
<td>$8 \times 10^{-12}$</td>
<td>$8 \times 10^{-13}$</td>
<td>$8 \times 10^{-14}$</td>
</tr>
</tbody>
</table>
OSCILLATOR MODEL

Fig. 1 Simplified block diagram of a quartz-crystal-controlled-oscillator. Loop gain = 1 and loop phase = $n2\pi n = 0, 1, 2, \ldots$.

FREQUENCY-TEMPERATURE-OVEN CHARACTERISTICS

Fig. 2. Idealized frequency versus temperature curve for a quartz resonator. The turnover point and the oven offset from turnover are indicated. From [16].
Fig. 3. Frequency versus temperature for a 5 MHz AT-cut resonator. The static temperature curve is shown in curve 1. The other curves show the response with \( \pm 4^\circ C \) sinusoidal temperature cycling at a sweep frequency of \( 9.2 \times 10^{-5} \) Hz-curve 2, \( 3.7 \times 10^{-4} \) Hz-curve 3, and \( 7.4 \times 10^{-4} \) Hz-curve 4. A model fit to the curves yields a dynamic temperature coefficient of \( -1.3 \times 10^{-5} \) s/\(^\circ C\). From [10].

Fig. 4. Frequency versus temperature for a 5 MHz SC-cut resonator. The static temperature curve is shown in curve 1. The other curves show the response with \( \pm 4^\circ C \) sinusoidal temperature cycling at a sweep frequency of \( 9.1 \times 10^{-4} \) Hz-curve 2, and \( 1.8 \times 10^{-3} \) Hz-curve 3. A model fit to the curves yields a dynamic temperature coefficient of \( 3 \times 10^{-7} \) s/\(^\circ C\). From [10].
Fig. 5. Frequency versus temperature for the primary timing clock of the Ginga satellite. From [33].

Fig. 6. Frequency versus the rf excitation level for a 5 MHz AT-cut resonator. From [19].
Frequency vs. Drive Level

Fig. 7. Frequency versus the rf excitation level for AT-, BT-, and several SC-cut resonators. The approximate power levels are indicated on the left margin. From [16, 33].

Frequency Change for in Plane Forces

AT-Cut

Fig. 8. Frequency shift versus diametrically opposed forces in the plane of the resonator as a function of the angle between the force and the azimuthal axis. From [28].
Fig. 9. Typical sensitivity of a AT-cut resonator to acceleration as a function of direction. From [19].

Fig. 10. Frequency of a precision oscillator as a function of orientation. Position 1 and 6 are the same. Note the slow change of frequency immediately after the rotation. This is likely due to small changes in the temperature gradient within the oscillator. From [23].
Fig. 11. Quiescent phase noise of a high quality 100 MHz oscillator and the induced phase noise from the application of sinusoidal acceleration at amplitude 2 g. An acceleration sensitivity of $2 \times 10^{-9}/g$ has been assumed.

Fig. 12. Frequency as a function of applied voltage. The slow variation after the change in voltage is due to the movement of ions within the resonator. From Gagnepain [1].
Fig. 13 Frequency change of quartz controlled oscillator due to a change in humidity. Fractional stability is approximately $2 \times 10^{-13}$ at 1 s. From [22].