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Phase noise measurements with a cryogenic power-splitter to minimize the cross-spectral collapse effect

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The cross-spectrum noise measurement technique enables enhanced resolution of spectral measurements. However, it has disadvantages, namely, increased complexity, inability of making real-time measurements, and bias due to the "cross-spectral collapse" (CSC) effect. The CSC can occur when the spectral density of a random process under investigation approaches the thermal noise of the power splitter. This effect can severely bias results due to a differential measurement between the investigated noise and the anti-correlated (phase-inverted) noise of the power splitter. In this paper, we report an accurate measurement of the phase noise of a thermally limited electronic oscillator operating at room temperature (300 K) without significant CSC bias. We mitigated the problem by cooling the power splitter to liquid helium temperature (4 K). We quantify errors of greater than 1 dB that occur when the thermal noise of the oscillator at room temperature is measured with the power splitter at temperatures above 77 K. *Published by AIP Publishing.* https://doi.org/10.1063/1.5006908

I. INTRODUCTION

One method of measuring the phase fluctuations of an oscillator (referred to as the Device Under Test—DUT) is to measure its deviations relative to a superior or similar performance reference oscillator.[1–5] When intrinsic noise of the readout system exceeds that of the DUT, a powerful measurement strategy known as the dual-channel cross-spectrum, as first suggested by Walls *et al.*,[6] is used. The DUT signal is shared between two phase sensitive channels, and the cross-spectral density of voltage fluctuations between their outputs is computed.[6–11] Since the intrinsic fluctuations of individual channels are not correlated, their contribution to the cross-spectrum decreases as $\sim 1/\sqrt{N_{avg}}$, where N_{avg} is the number of averages taken by the Fast Fourier Transform (FFT) analyzer. This technique of cross-spectrum, however, is susceptible to experimental errors and inconsistencies that can result in over- or under-estimation of the DUT noise.[12–19] The sources of noise over-estimation were understood, but the mechanism of the noise under-estimation was not clearly known until recently.[17] In recent years, severe noise under-estimation due to the "cross-spectral collapse" (CSC) effect has become more visible as more state-of-the-art oscillators' noise approaches their thermal noise limit. The source of this effect is often the power splitter such as the four-port hybrid coupler and Wilkinson power splitter (WPS). The thermal noise fluctuations originating from the termination resistor of the hybrid coupler or the isolation resistor of the WPS appear anti-correlated (phase-inverted) between two outputs of the splitter.[12,20–23] In this case, the cross-spectrum measures the differential thermal noise between the power splitter and the DUT[12] and grossly underestimates the white-noise level. Not only do reactive power splitters cause spectral collapse but different types of resistive power splitters can also introduce partial or full thermal noise collapse when amplifiers or isolators are added to their outputs to increase isolation.[18]

The goal of the work reported here is to accurately measure the phase noise of a thermally limited oscillator at room temperature using the cross-spectrum method. We reduce the bias due to the CSC effect by cryogenic cooling of the reactive power splitter. A non-cryogenic solution to prevent undesired CSC effects was discussed earlier that uses a modified reactive power splitter at room temperature.[24] Although the preliminary result is promising, the technique can be difficult to implement in practice.

II. PHASE NOISE MEASUREMENT OF THERMALLY LIMITED OSCILLATOR

A. Experimental setup

Figure 1 shows a schematic diagram used for the phase noise measurement of the DUT. The DUT includes an oscillator and an output attenuator "A." A pair of phase-sensitive detectors operates simultaneously to comprise the basic cross-spectrum phase noise measurement. The DUT signal goes through an impedance matched, non-reflective harmonic filter (IMHF), a calibrated additive white phase noise source (AWNS), and then to a 3-port reactive power splitter (PS) that is mounted inside of a cryostat. Each output of the power splitter feeds separate, optimized single-channel phase detectors measuring the relative phase fluctuations between the DUT and separate oscillators (Ref #1 and Ref #2, as shown in Fig. 1) using phase locked loops (PLL) to maintain phase quadrature at the doubled balanced mixers (DBM1 and DBM2). The phase fluctuations at the mixer inputs convert to output voltage fluctuations at baseband frequencies (f) above the PLL bandwidth. The two-channel FFT analyzer

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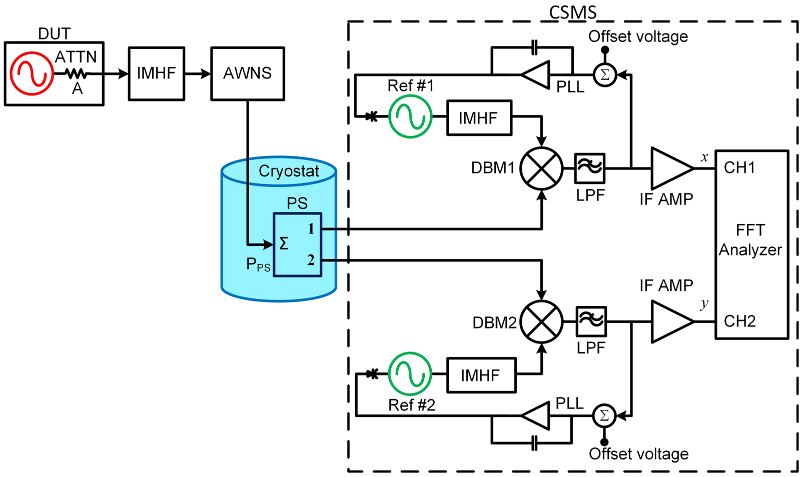

FIG. 1. Block diagram of a conventional dual-channel cross-spectrum system used for measuring phase noise of the DUT. The FFT analyzer measures the correlated spectrum which consists of the phase noise of the DUT, the anti-correlated noise of the reactive power splitter (PS), and the uncorrelated noise from separate detector channels which averages down toward zero, lowering the measurement system's overall noise floor. DBM—Double Balanced Mixer, LPF—Low Pass Filter, IMHF—Impedance Matched Harmonic Filter, AWNS—Additive White Noise Source, IF AMP—Intermediate Frequency (baseband) amplifier, CSMS—Cross-Spectrum Measurement System.

then computes the cross-power spectral density (S_{yx}) between inputs x and y of channels 1 and 2. The AWNS is used to determine the mixer sensitivity. A variable dc offset voltage is added at the input of the PLL integrator to minimize the amplitude modulation (AM)-to-phase modulation (PM) conversion in the mixers. The AM-to-PM conversion of the phase noise measurement system was suppressed by more than 30 dB for different operating conditions of this experiment; this was verified by introducing AM sidebands of known level.

The measurements were taken in a variable temperature cryogenic system. The system has a liquid helium Dewar and feedback controlled sample temperature from 1.8 to 350 K with a stability of ±0.01 K. The space for the sample is shielded in a superconducting environment and is configured with three stainless steel RF coaxial cables. These cables were connectorized with SubMiniature version A (SMA) plugs to allow frequencies up to 26 GHz. We used an available NIST cryogenic system that requires a superconducting shield for other physical property measurements. However, this shield is not necessary for the phase noise measurements. The power splitter used for this experiment is shown in Fig. 2(a), and the sample fixture is shown in Fig. 2(b). Shown on the right [Fig. 2(c)] is the physical apparatus of the basic cross-spectrum phase noise measurement as shown in Fig. 1.

When a 3- or 4-port reactive power splitter is used, the cross-spectrum measurement system forms a differential temperature measurement and it is proportional to ($T_{\mathrm{DUT}} - T_{\mathrm{PS}}$) in the thermal noise region.[12,19] Here, T_{DUT} and T_{PS} are the effective noise temperatures of the DUT and the power splitter's resistance, respectively. The measured double sideband (DSB) phase noise, $S_\varphi(f)_{Measured}$, is expressed as

$$S_\varphi(f)_{Measured} = S_\varphi(f)_{DUT} - \frac{k_B}{P_{PS}}(T_{PS}), \tag{1}$$

where k_B is the Boltzmann constant. Equation (1) contains the true DUT noise and the anti-correlated thermal noise of the power splitter. When the thermally limited DUT and power splitter are at the same temperature, the measurement bias due to the power splitter is almost equal to the DUT noise. This causes thermal noise collapse and results in a measured noise level far below the true value. The estimated bias can be removed numerically in post-processing;[19] however, the removal of the source of a measurement bias is typically superior to the numerical post-measurement compensation of the said bias. Although it may not be a practical solution, we have demonstrated that the bias from the thermal noise of the power splitter can be reduced by lowering its temperature. When the power splitter is kept at liquid helium (4 K) temperature, its

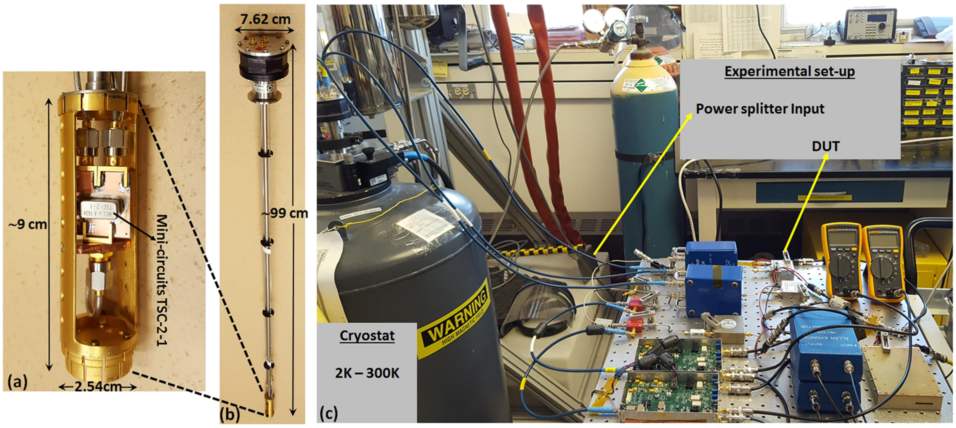

FIG. 2. (a) Picture of the commercial-off-the-shelf (COTS) power-splitter that is carried by the mounting fixture which is inserted into the cryostat, (b) mounting fixture, and (c) experimental setup used for the measurement of phase noise of the DUT.

thermal noise is 18.8 dB lower than the DUT thermal noise at 300 K.

B. Characterization of the power splitter

The power splitter for the measurement was chosen based on its good port-to-port isolation, low loss at 50 Ω terminating impedance, and small size ensuring that it fits the cryostat's mounting fixture. A suitable choice was the Mini-circuits TSC-2-1 (Commercial manufacturer is indicated for information purposes only. Other manufacturers exist. No endorsement is implied) and was not particularly hand-selected among several. From physical dissection and information from the manufacturer's website, the splitter's structure was determined to be a reactive double tapped auto-transformer splitter. It is implemented with impedance transformers and a nearly 100 Ω isolation resistor as shown in Fig. 3(a). For all practical purposes relevant to this experiment, it produces anti-correlated thermal noise identical to that of the Wilkinson power splitter analyzed before.[18] Figure 2(a) shows the placement of the power splitter on a small printed circuit board (PCB) carrier. It was connectorized with three adaptors to interface SMA-fitted stainless steel semi-rigid coaxial cables. Relevant S-parameters (transmission and isolation) were measured as a function of coarse high temperature with fine-temperature measurements near 4 K. Figure 3(b) shows that the transmission between port 1 and port 2 (S_{21}) was nearly constant over the full temperature range. Results for S_{31} were similar to that of S_{21}. Isolation of the outputs was measured by the power transfer between output ports 2 and 3 (S_{23}). It assures that the equivalent input noise of the mixer in one channel ["(DBM)" in Fig. 1] does not couple into the other channel enough to cause a measurement bias. Figure 3(b) shows that the output-port isolation remained at 27 dB or higher, more than adequate, and reducing only by about 7 to 8 dB and only at temperatures below 10 K. The loss between input and outputs ports intrinsic to the power splitter without the stainless steel coaxial cables was 3.3 dB. The S-parameters shown in Fig. 3 include the cable loss of approximately 1.1 dB.

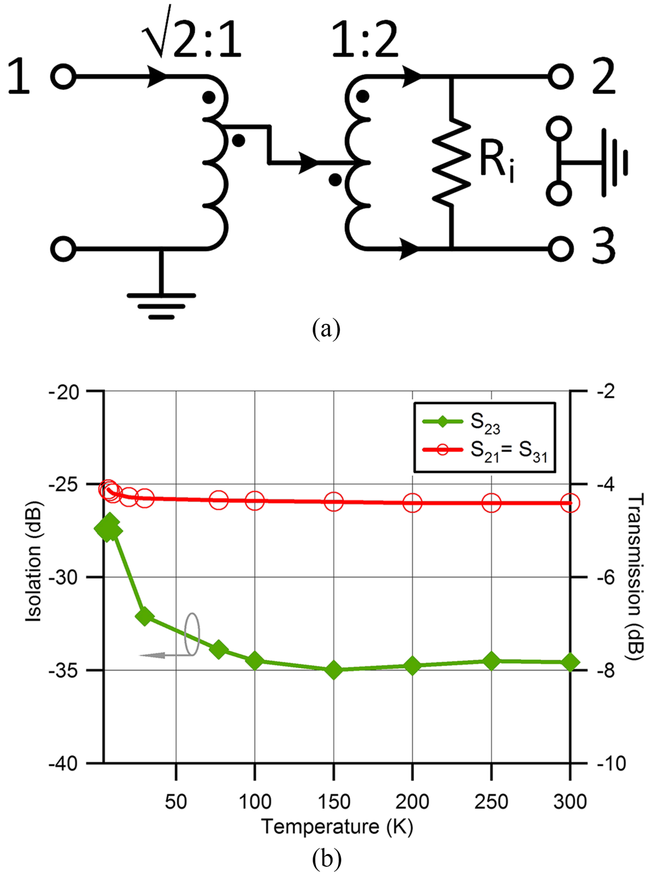

FIG. 3. (a) Schematic of the double tapped auto-transformer based reactive power splitter. Isolation resistor, $R_i \sim 100\ \Omega$. (b) Variation of isolation (S_{23}) and transmission (S_{21} and S_{31}) of the power-splitter with temperature at 100 MHz.

C. Phase noise results

Phase noise measurements were carried out on a commercial ultra-low noise quartz crystal oscillator operating at 100 MHz as the DUT.[18,25] The principle and design of such oscillators for ultra-low thermal noise are described by Rohde[26] and Gruson *et al.*[19] Note that the DUT includes a resistive attenuator "A" at the output of the oscillator assuring a nominal 50 Ω output impedance match and defining the thermal noise. The phase noise of the DUT was measured with net power "P_{PS}" into the power splitter (Fig. 1) equal to +10.4 dBm. Thus, the expected thermal-limited DSB phase noise of the DUT is given by the ratio of wideband (Johnson) thermal noise to carrier power, or $10\log(k_B T/P_{PS}) = -184.4$ dBrad^2/Hz. Referring to Fig. 4, the measured white noise level at 300 K using the cross-spectrum method is the red plot that is predominantly well below −184.4 dBrad^2/Hz, by at least 10 dB, a clear indication of spectral collapse. When the splitter was cooled to 4 K, the spectral-collapse disappeared and the measured phase noise as shown by the blue

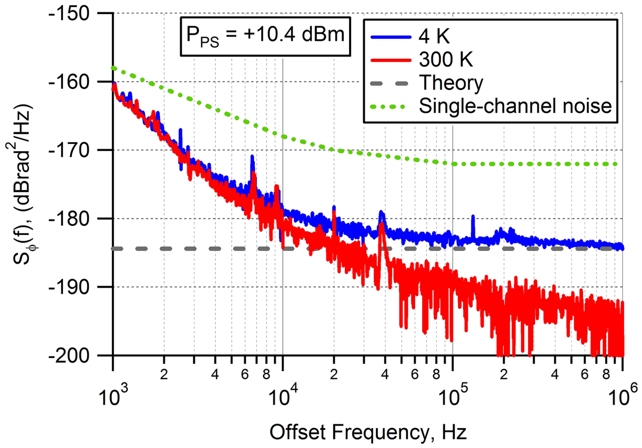

FIG. 4. Phase noise of a 100 MHz oscillator measured with a Wilkinson power splitter at 300 K and 4 K. The theoretical thermal noise of this oscillator referenced to the input power of the common-mode power splitter (P_{PS}) is −184.5 dBrad^2/Hz. The measured noise at 4 K matches the theoretical value; however, there is a clear indication of spectrum collapse at 300 K, as expected. Each channel of the cross-spectrum measurement system has nearly same noise. The number of FFT averages (N_{avg}) for different frequency spans is as follows: 1 kHz–10 kHz = 5000, 10 kHz–30 kHz = 17 000, 30 kHz–100 kHz = 50 000, 100 kHz–300 kHz = 170 000, and 300 kHz–1 MHz = 500 000. The same number of FFT averages is used for all the phase noise results shown in this paper.

curve in Fig. 4 agrees with the theory. This is the first experimental demonstration of the effective elimination of the CSC effect by cryogenically cooling the power splitter. The phase noise results presented in this paper are obtained from the biased magnitude estimator, $|\langle S_{yx}\rangle|$. An excellent analysis of different types of biased and unbiased estimators is described by Rubiola *et al.*[19,27]

To further verify that the DUT thermal noise measurements agree with the theoretical value, we varied the input power of the power splitter (P_{PS}) while recording the phase noise. When the thermally limited DUT and power splitter were at the same room temperature, the measurement caused spectral collapse, resulting in a value far below the theoretical value at any P_{PS}. However, when the splitter temperature was 4 K, the measurement was in close agreement with the theory (within ±1.1 dB 2-sigma measurement uncertainty) for different P_{PS}. Figure 5 shows measured noise of the DUT at +2.4 dBm, +6.4 dBm, and +10.4 dBm that agrees with the theoretical value of −176.4 dBrad^2/Hz, −180.4 dBrad^2/Hz, and −184.4 dBrad^2/Hz, respectively. Additionally, when the power splitter is at liquid helium (4 K) and liquid nitrogen (77 K) temperatures, thermal noise of its isolation resistor is 18.8 dB and 5.9 dB, respectively, lower by these amounts than the DUT's thermal noise at 300 K. Therefore, the cross-spectrum measurement should introduce an error of −0.1 dB and −1.3 dB, respectively, at these two temperatures. As verification, measurements were performed by varying the power splitter temperature while maintaining a fixed P_{PS}. Figure 6 summarizes the phase noise measurements at four different splitter temperatures; it shows varying amounts of the CSC effect that bias the measurement of thermal noise of the DUT. We observed collapse errors of approximately −1.2 dB at 77 K and −4.2 dB at 200 K, close to the theoretical prediction and within ±1.2 dB 2-sigma measurement uncertainty. The phase noise of the DUT at 700 kHz offset frequency and the corresponding measurement error due to the CSC effect are shown in Fig. 7 for P_{PS} equal to +10.4 dBm. The results in Fig. 6 also confirm that a cross-spectrum measurement system forms a differential temperature measurement between the DUT and the power splitter. This also validates the findings by Ivanov *et al.*[12] and Gruson *et al.*[19]

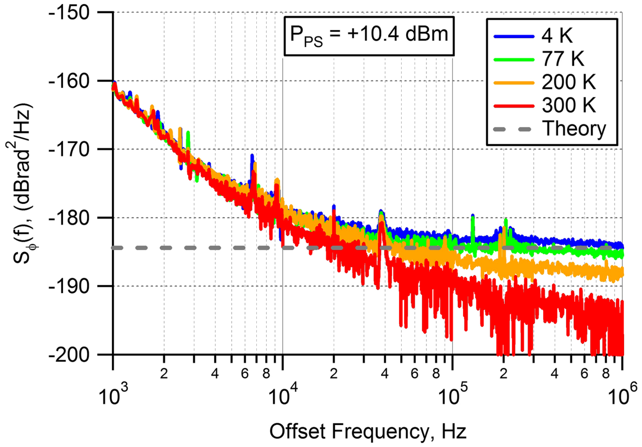

FIG. 6. Phase noise of a 100 MHz oscillator with varying power splitter temperature for a constant P_{PS}. Full collapse (limited by the number of FFT averages) at 300 K and partial collapse at 77 K and 200 K are observed as expected.

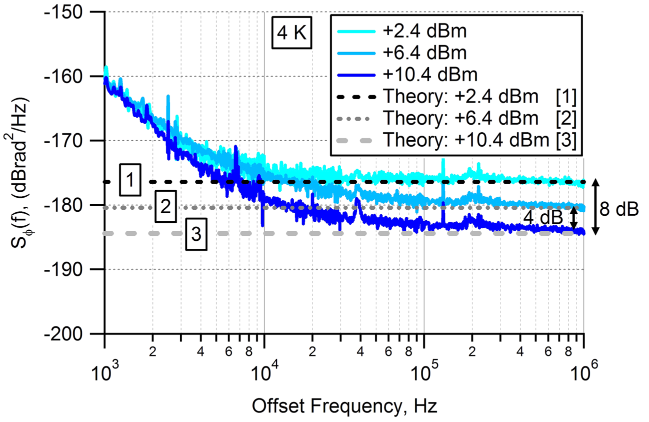

FIG. 5. Phase noise of a 100 MHz oscillator measured for different input powers of the power splitter (P_{PS}) while the temperature of the power splitter is kept constant at 4 K. The measured white noise levels agree with theoretical values of thermal noise at +2.4 dBm, +6.4 dBm, and +10.4 dBm as −176.4 dBrad^2/Hz, −180.4 dBrad^2/Hz, and −184.4 dBrad^2/Hz, respectively.

Finally, to verify that the thermal fluctuations generated inside the transmission line connecting the DUT at room temperature to the cold power splitter in the cryostat are not affecting the measurement results, we performed the test shown in the inset of Fig. 8. Both the DUT and the power splitter were kept at room temperature; a coaxial cable approximately 200 cm long was routed to and from the cold spot inside the cryostat. The length of the cable was almost twice the length of the common mode cable used to deliver the DUT signal to the power splitter in Fig. 1. In principle, the thermal fluctuations of the DUT and power splitter at room temperature should cancel each other in the cross-spectrum which means we should see the residual thermal fluctuations of the cable above 50 kHz offset frequency. The temperature of the cable was varied from 4 K to 300 K, and the results for each temperature are shown in Fig. 8 for P_{PS} = +9.5 dBm. This finding indicates that at 4 K, 77 K, and 200 K, the thermal fluctuations of the cable have negligible effect on the differential thermal

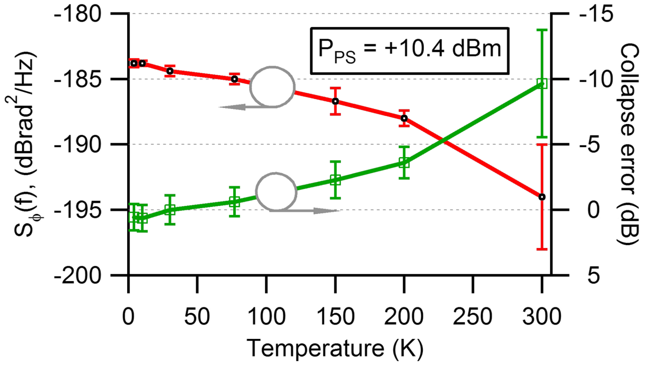

FIG. 7. Variation of the phase noise of a 100 MHz oscillator as a function of temperature at offset frequency f = 700 kHz. The secondary (right) axis depicts the error due to the cross-spectrum collapse. Each data point is obtained by averaging 40 noise data points above and below the 700 kHz offset frequency. Accurate measurements within ±1.1 dB 2-sigma measurement uncertainty were observed for temperatures below 30 K.

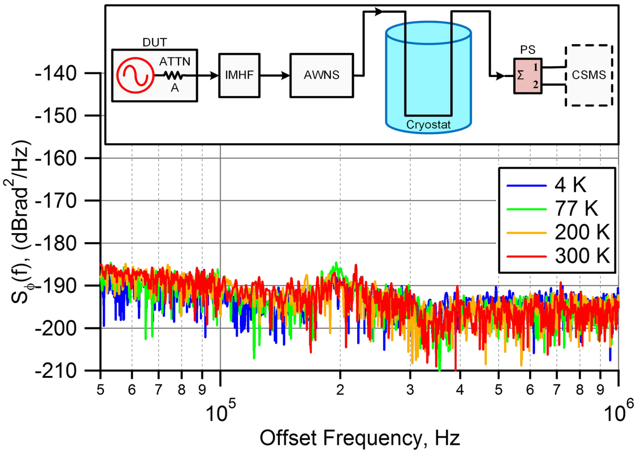

FIG. 8. Variation of residual phase noise of the coaxial cable with temperature. Schematic of the measurement setup is shown in the inset; the cross-spectrum measurement system (CSMS) is the same as in Fig. 1 The thermal noise of the DUT and the power splitter cancels each other at room temperature, and this measurement scheme provides residual phase noise of the cable at offset frequencies above 50 kHz. Below 50 kHz, the absolute phase noise of the oscillator dominates. IMHF—Impedance Matching and Harmonic Filtering, AWNS—Additive White Noise Source.

noise measurements between the DUT and the power splitter. Thus, the phase noise results shown in Figs. 4–6 are not biased by the cable thermal fluctuations.

III. SUMMARY

Cross-spectral analysis is a commonly used method for increasing the sensitivity of phase noise measurements. The advantage of this method is that by splitting the signal from a single DUT to drive two measurement channels, noise from components in each channel is uncorrelated between the two channels, such that with sufficient averaging the noise converges to that of the DUT. However, as the power splitter itself is common to both measurement channels, anti-correlation of the thermal noise of its isolation resistor can introduce significant errors when measuring a phase noise of a thermally limited oscillator as the DUT. This occurs because the cross-spectrum performs an undesired differential temperature measurement between the DUT and the splitter. This paper shows the first experimental proof that the theoretically expected levels can be measured with essentially no bias or collapse-error present by operating the reactive power splitter at 4 K in a cross-spectrum measurement. We further confirm that errors of greater than 1 dB result when the DUT at room temperature is measured with a power splitter at temperatures above 77 K. In conclusion, when both the thermally limited DUT and the reactive power splitter are at the same temperature, the CSC effect due to anti-correlation dominates; however, an accurate measurement of the DUT noise is possible by lowering the temperature of the power splitter.

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