Cross-spectral Collapse from Anti-correlated Thermal Noise in Power Splitters

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***Abstract*—We discuss the cross-spectral collapse due to anti-correlated thermal noise that originates from the common-mode power divider (splitter) in a cross-spectrum noise measurement system. We studied this effect for different power splitters and discuss its influence on the measurement of thermal-noise limited oscillators.**

Keywords—anti-correlation; cross-spectrum; oscillator; phase inversion; phase noise; power spectral density; thermal noise

I. INTRODUCTION

The cross-spectrum technique is a standard procedure used for the measurement of phase modulation (PM) and amplitude modulation (AM) noise of oscillators [1]–[4]. Despite being commonly used, this technique is susceptible to experimental errors [5], [6]. One among many of these errors is caused either by positive or negative correlations resulting in over-estimation or under-estimation respectively of the oscillator noise [7], [8].

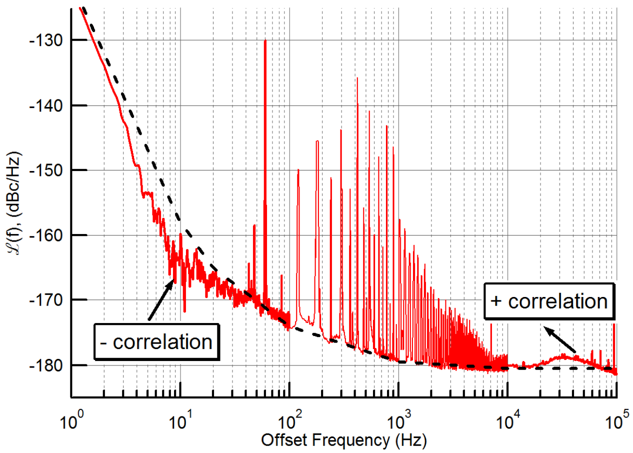

Fig. 1. Phase noise of a 5 MHz oscillator. The red curve shows the effect of positive and negative correlation on the phase noise. The curve in black is the actual noise of oscillator.

An example of such effect on the phase noise is illustrated in Fig. 1. Here, the red curve is the measured phase noise of a 5 MHz quartz-crystal oscillator; it displays simultaneously the effect of both positive correlation above 10 kHz offset and negative correlation around 10 Hz offset. One cause of the noise under-estimation as explained in [8] can occur due to the anti-correlation (phase-inversion) collapse mainly from AM noise leakage. More recently, a different source of anti-correlation in a cross-spectrum measurement has been identified; its origin is from the common-mode power splitter [9]–[11]. Correlated thermal noise of the power splitter appears equally but in opposite phase in two channels of the cross-spectrum system. In this paper, we will discuss the effect of thermal noise of various reactive and resistive power splitters on the noise measurement of thermally-limited oscillators.

II. CROSS-SPECTRAL COLLAPSE DUE TO POWER SPLITTERS

In recent years, several commercial ultra-low phase noise (ULPN) oscillators have been introduced whose phase noise is near the thermal limit. In this new class of oscillators, a bias (either positive or negative) from the power splitter thermal noise can dominate the cross-spectrum result. In order to study the effect of thermal noise of the common mode power splitter on absolute noise measurements, a selection of reactive and resistive power splitters as shown in Fig. 2 were tested with an ULPN oscillator at 100 MHz [Golden Citrine from Wenzel Associates, Inc.].

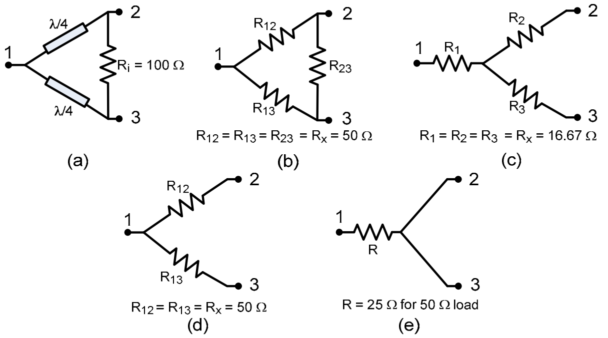

Fig. 2. Schematic of different power splitters: (a) Wilkinson power splitter, (b) Resistive 3-R (Delta configuration), (c) Resistive 3-R (Wye configuration), (d) Resistive 2-R and (e) Resistive 1-R.

The measurement set-up as shown Fig. 3 is used for the phase noise measurement of this oscillator. A variable dc offset

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voltage was added at the input of the phase locked loop (PLL) integrator to reduce the device under test (DUT) AM noise thus minimizing the effect of anti-correlation collapse due to the AM noise leakage [8]. At first a Wilkinson power splitter (WPS) was chosen and the phase noise of the oscillator was measured with attenuator 'A' equal to 3 dB at the output of the oscillator. Assuming a 50 Ω system and taking into account the loss of DUT signal strength in the impedance matching and harmonic filtering (IMHF) circuit, the theoretical noise of a thermally-limited oscillator should be -188.5 dBc/Hz i.e., -177 - P_{PS}. As shown in Fig. 4, a complete collapse (limited by the number of FFT averages) of the thermal noise spectrum was observed due to the anti-correlated noise of the WPS. Next, the measurement was repeated with approximately 9 dB attenuation of the test signal; instead of the theoretical noise level of -182.5 dBc/Hz we again achieved a cross-spectrum collapse. The power levels at the LO and RF ports of the phase detector in each channels were kept constant for both 3 dB and 9 dB measurements.

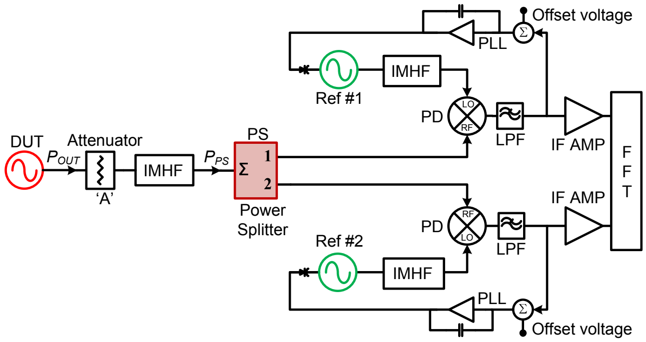

Fig. 3. Block diagram of a cross-spectrum phase noise measurement system. DUT – Device under test, IMHF – Impedance matching and harmonic filtering, LPF – Low pass filter, PD – Phase Detector, FFT- Fast Fourier Transform, PLL – Phase Locked Loop.

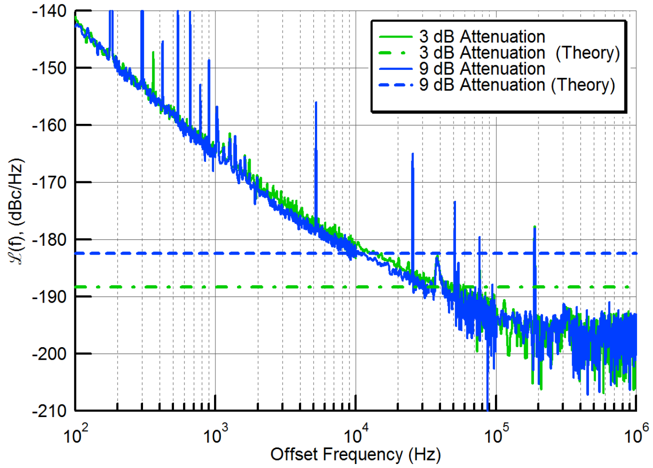

Fig. 4. Phase noise of a 100 MHz oscillator measured with a Wilkinson power splitter. Theoretical noise of this oscillator referenced to the input power of common-mode power splitter (P_{PS}) is respectively –188.5 dBc/Hz and -182.5 dBc/Hz for attenuator 'A' equal to 3 dB and 9 dB. These values are calculated from (–177 –P_{PS}) assuming a 50 Ω system. The far-from-the-carrier noise in both cases are limited by the number of FFT averages (N = 100,000) but there is clear indication of a spectrum collapse.

It is especially worth noting that for 9 dB attenuation in Fig. 4, one can actually extract underlying multiplicative sloped noise of the oscillator between 10 kHz and 50 kHz offset frequencies even though it is normally masked by the thermal noise. This observation validates the power-law noise model [12], [13].

The PM noise of the same oscillator was also measured with 3-R, 2-R and 1-R resistive power splitters (Fig. 2(b)-(e)) and the results are shown in Fig. 5. The measured thermal noise is almost 3 dB lower than the simulated noise levels (Table 1) in each case. This discrepancy is due to the fact that the simulations are performed with an ideal 50 Ω load impedance; however, in actual experiment the power splitter is connected to the reactive load of the double balanced mixer used as a phase detector. In spite of the discrepancy between the measurements and simulation, the relative differences in the measured thermal noise levels between 1-R and 3-R as well as between 3-R and 2-R power splitters are nearly the same as the simulated results. In [11], we demonstrated that if the power splitter is connected to a real 50 Ω load impedance, the simulation and experimental results agree well; we verified this for the AM noise measurements.

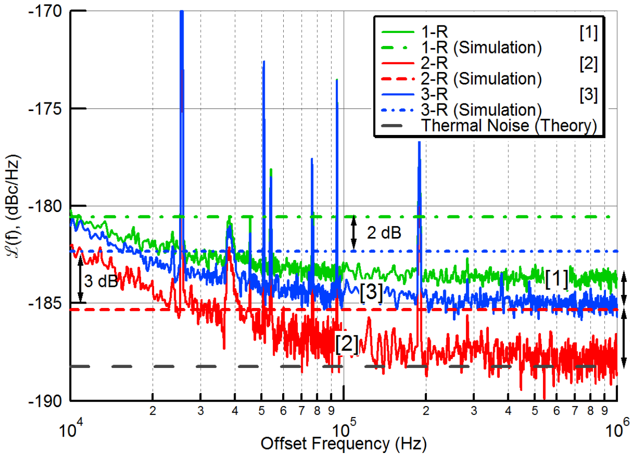

Fig. 5. Phase noise of a 100 MHz oscillator measured with resistive 1-R, 2-R and 3-R power splitters. The simulated noise for three different power splitters is obtained from Table 1, column 8.

The simulation results given in Table 1 for different types of power splitters were performed in the Advanced Design System (ADS) software. The block diagram for the simulation is shown in Fig. 6. For this simulation, the thermal noise contribution of the source, the power splitter and the load resistors are considered. A detailed description of the simulation can be found in [11]. Table 1 tabulates the results of thermal noise contribution of the individual component to the output cross-spectrum as a fraction of the noise from R_s. The simulation is performed for load and source impedances equal to 50 Ω and at 300 K temperature. The values reported in the table are from the expected value of the cross-spectrum. All uncorrelated cross-terms, which reside in the imaginary component of the cross-spectrum, are zero and the result is an entirely real number. For an exact measurement of the thermal noise of an oscillator, columns 3 and 8 should be equal.

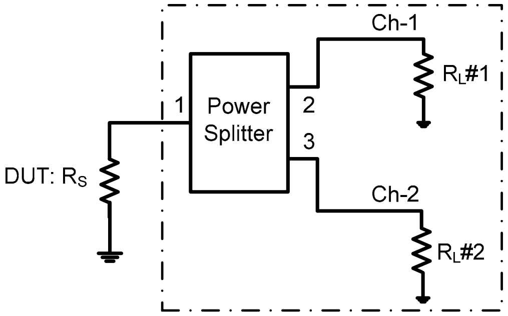

Fig. 6. The main sources of thermal noise used for ADS simulation. R_s represents the thermal noise of the source or the device under test (DUT). Total number of resistors in the power splitter varies from 0 to 3 depending on the configuration (see Fig. 2). The load resistors R_L#1 and R_L#2 represent the thermal noise of the measurement system.

Table 1: 2-way Power Splitter (PS): Source impedance (Z_S) = Load Impedance (Z_L) = 50 Ω, T = 300 K, Isolator: Insertion Loss = 0 dB, Isolation = ∞

1	2	3	4			5	6	7	8
Case #	*Type of Power splitter (PS)*	*Relative cross-spectrum of individual component* $S_{Ch2-Ch1}(f)/S_{R_s}(f)$						*Total Noise*	
		R_s	*Power Splitter* R_1	R_2	R_3	*R_L#1*	*R_L#2*	*w/o* R_s	*All Components*
1	Wilkinson R_i = 100 Ω	1	-1			0	0	-1	0
2	3-R Wye R_x = ~17 Ω	1	1/3	-2/3	-2/3	2	2	3	4
3	3-R Delta R_x = 50 Ω	1	0	-1	0	2	2	3	4
4	2-R R_x = 50 Ω	1	0	-3/4	-3/4	5/4	5/4	1	2
5	1-R, R = 25 Ω	1	1/2	-	-	9/5	9/5	5	6

Here, R_i and R_x respectively correspond to the isolation resistor and the resistors for 2-R and 3-R power splitters.

The simulation results in Table 1 indicate that the 3-R (Delta or Wye) and 2-R splitters produce anti-correlated thermal noise between the outputs. Also, the resistive power splitters do not have sufficient isolation to allow a cross-spectrum measurement to overcome the loss of signal-to-noise ratio in each individual channel. They cannot be used to accurately measure a thermally limited source because the dominating noise of the load to the power splitter appears correlated in both channels and cannot be rejected.

We also observed large variations in the measured thermal noise as shown in Fig. 7 when different phase detectors were used. The dissimilar reactive load presented by different phase detectors to the power splitter affects the degree of anti-correlation.

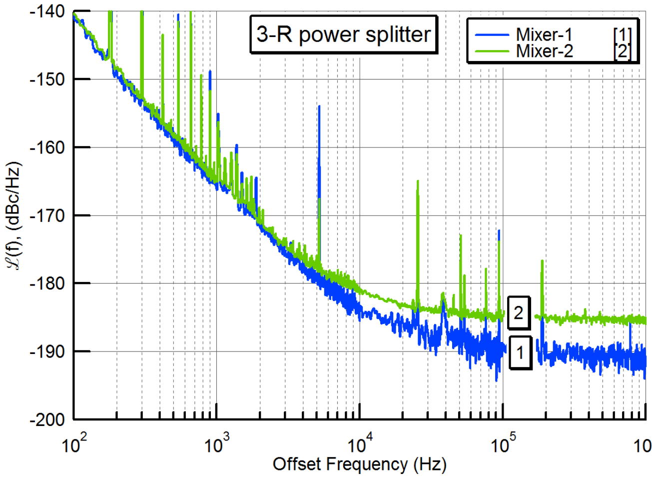

Fig. 7. Phase noise of a 100 MHz oscillator measured with a resistive 3-R power splitter but different mixers (phase detectors) with different input impedances.

III. CONCLUSION

We discussed cross-spectral collapse due to anti-correlated thermal noise of the common-mode power divider (splitter) in a cross-spectrum noise measurement system. The simulation and experimental results indicate that the reactive and resistive power splitters cannot accurately measure a thermal-noise limited oscillator using the scheme in Fig. 3, if the oscillator and power splitter are at the same temperature.

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