CHAPTER 9

THE NATIONAL BUREAU OF STANDARDS ATOMIC TIME SCALE: GENERATION, STABILITY, ACCURACY AND ACCESSIBILITY*●

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"Go wond'rous creature, mount where science guides,
Go, measure earth, weigh air and state the tides;
Instruct the planets in what orbs to run,
Correct old time, and regulate the sun,"

Pope's *Essay on Man*

The atomic time scale at the National Bureau of Standards, AT(NBS), depends upon an ensemble of continuously operating cesium clocks calibrated occasionally by an NBS primary frequency standard. The data of frequency calibrations and interclock comparisons are statistically processed to provide near-optimum time stability and frequency accuracy. The noise spectrum of each clock is represented by a simple mathematical model, with parameters determined by the behavior of that clock. These noise parameters are used in a nearly optimum procedure for periodically recalibrating the frequency of each clock and for combining the clock readings to produce AT(NBS). The long-term fractional frequency stability of AT(NBS) is estimated to be a few parts in 10^{13}, and the accuracy is inferred to be 1 part in 10^{12}.

A small coordinate rate is added to the rate of AT(NBS) to generate UTC(NBS): this small addition is for the purpose of maintaining synchronization within a few microseconds of other international timing centers. Today, UTC(NBS) is operationally available over a large part of the world via; WWV, WWVH, WWVB, and telephone; some time transfer systems, e.g., Loran-C and the TV line-10 system; and experimental systems such as the ATS–3 satellite. We indicate the precision and accuracy of these dissemination systems.

The clocks composing AT(NBS) provide part of the input into the International Atomic Time scale (TAI). The TAI scale is described and new proposals for improvement of this time scale are discussed. Many of the concepts and algorithms developed in this chapter may be directly applicable to the construction of a TAI scale in conformity to the SI unit of time.

Key words: AT(NBS); atomic clock; atomic time scale; clock ensemble; primary frequency standard; SI second; TAI; time/frequency dissemination; time scale; time scale algorithms; UTC(NBS).

9.1. INTRODUCTION

Although the atomic definition of the Système International (SI) Second has been accepted internationally, several differing techniques have been advanced for the construction and maintenance of atomic time scales. These varied techniques capitalize to different degrees on such factors as accuracy, stability, modeling and simulation of atomic clocks, availability and cost and, to a large extent, where developed within particular circumstances of given laboratories. Such techniques include a time scale averaged over a multiplicity of selected commercial cesium beam clocks [1],[1] continuous use of a long-beam, primary cesium standard (laboratory) for generation of a time scale more nearly in conformity with SI [2], maintenance of a time scale generated by an ensemble of statistically weighted cesium atomic clocks which are periodically compared to an evaluable primary frequency standard (laboratory) [3, 4], and establishment of a statistically weighted atomic time scale based on some 7 International atomic time scales (International Atomic Time – TAI) [5]. The independent atomic time scale at the National Bureau of Standards, AT(NBS), depends upon an ensemble of continuously operating cesium clocks calibrated occasionally by an NBS primary frequency standard from which the AT(NBS) scale derives its accuracy. The stability of the ensemble between calibrations is of fundamental importance.

The instabilities of each clock in the ensemble may be bicategorized: First, there are deterministic processes that should be considered for each clock; e.g., frequency and time offsets, changes in these offsets, and frequency drift. Changes or drifts in frequency may be estimated by referring to the definition of time across the ensemble and/or with reference to a primary standard. Second, there are random fluctuations (nondeterministic). The noise spectrum of these random fluctuations for each clock is deduced by comparing each clock with all the others. A simple mathematical model reasonably represents this noise spectrum with parameters determined by the random behavior of each clock. These noise parameters provide near optimum filtering of each clock's noise and give a best estimate (in the sense of minimum squared error of prediction) of the apparent time and frequency of each clock with respect to the clock ensemble. Knowledge of the noise spectrum for each clock allows an estimate of the noise of the ensemble; the long-term random fluctuation of the fractional frequency of AT(NBS) are estimated to be a few parts in 10^{14}, whereas, the instability due to deterministic process are estimated to be less than about 2×10^{-13}/year.

The inaccuracy of a primary frequency standard may also be bicategorized. In an evaluation of the parameters which affect the frequency of the primary standard, there are two factors associated with each parameter; i.e., a bias (possibly zero) and a random uncertainty in our knowledge of its effect. If the time scale has excellent stability, one can average the random portion (in an appropriate weighted sense) of all the frequency calibrations with a primary frequency standard.

The AT(NBS) scale in overview is an ensemble of eight commercial cesium beam clocks maintained independently. The clocks are statistically weighted (i.e., filtered) to generate a time scale, AT(NBS), with nearly optimum stability. This scale is used as a memory for frequency in utilizing all of the frequency calibrations with respect to an NBS primary frequency standard. These calibrations are then used after appropriate weighting and filtering to determine the proper[2] rate and the accuracy of the AT(NBS) scale. This scale, along with the atomic time scales of six other laboratories, is used to generate the International Atomic Time Scale, TAI, at the Bureau International de l'Heure (BIH) [5].

In conjunction with the AT(NBS) proper time scale, we also generate the coordinate time scale, UTC(NBS). This latter scale is both synchronized (coordinated) to within a few microseconds of the UTC(BIH) scale and mutually coordinated with the UTC(USNO) scale. This coordination is accomplished by small discrete rate changes (of the order of 10^{-13}) in UTC(NBS) and in UTC(USNO). One second time jumps are made as announced by the BIH for keeping these scales within 0.7s of the UT1 scale [6] (see chap. 1).

The UTC(NBS) scale is used as the reference for time and frequency broadcasts of the National Bureau of Standards. The time of this scale and frequencies derived therefrom are currently made available via sundry methods: e.g., the two radio transmitters at Ft. Collins, Colo., WWV and WWVB [7, 8]; the radio transmitters at Kekaha, Kauai, Hawaii, WWVH; portable clocks [9]; both the television color subcarrier and line-10 time transfer systems [10, 11]; telephone (303) 499–7111; and the experimental ATS–3 satellite [12], which broadcasts time and frequency information with a format similar to that of WWV and WWVH. The future holds many possibilities of providing time and frequency information including an active TV line-21 system, a relay satellite system, and a time code on the Omega transmissions.

The chapter discusses basic ideas inherent in consideration of an atomic time scale including terminology, statistical clock modeling, and realization of a near optimum atomic time scale. The discussion includes topics such as deterministic

[1] Figures in brackets indicate the literature references at the end of this chapter.

[2] Proper is used here in the relativistic sense.

properties, random perturbations of a clock's time, and atomic time scale algorithms. There is a detailed description of the NBS atomic time scales including their derivation and stability. Also, there is a brief discussion of the TAI and its stability; the ensemble analysis has relevance to the composite International Atomic Time (TAI) scale. The chapter includes a description of the UTC(NBS) time scale, its accessibility, and a comparison of accuracy, coverage, cost, etc. for different methods of access. The annexes at the end of the chapter give detailed derivations and an example of a computer program of a time scale algorithm.

9.2. BASIC TIME AND FREQUENCY CONSIDERATIONS

At this point we will review some basic time and frequency ideas described in other chapters of this monograph. Time—as a fundamental parameter in physics—almost always appears as an independent variable in any of the physical laws involving time. In contrast; it is our intent to derive *time*, i.e., we will express time as a dependent variable. Explicitly, time will be a function of how a particular clock ensemble and frequency standard are utilized. (A *clock* is a frequency standard coupled to a divider or counter.) This functional dependence may clearly be formalized by introducing some fundamental concepts of time.

First is the concept of *time interval*. Currently, the SI unit of time, the second, "is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom" as defined by the General Conference of Weights and Measures [13]. Since the frequency of the radiation corresponding to the above transition, ν, has also been defined as 9 192 631 770 Hz, we retain the convenient relationship that $\nu_I = 1/\tau_I$, and we see simply how a frequency standard is also a time interval standard. The second concept is that of *date*, clock reading, or clock time which often has been called epoch.[3] Date is simply the counting or accumulation—starting from some predetermined origin—of unit time intervals.

Explicitly, the date, t, is a counting of the periods of the above defined cesium transition; i.e.,

$$t = N_I \tau_I + t_0, \tag{9.1}$$

where t_0 denotes some defined and/or agreed upon date of an event when the counting started, N_I is the number of periods that have occurred since t_0, and τ_I is the ideal period given by the defined cesium resonance. The ideal proper time of a clock is, therefore, given by t in eq (9.1).

[3] See discussion of date and epoch in Chapter 1.

A third concept is *simultaneity:* Two events are simultaneous if equivalent signals, propagating in a given media arrive coincidentally at a common point in space which is geometrically an equal distance from the source of each event. In practice a much broader definition is often used for clock synchronization; i.e., two clocks have the same reading in a specific reference frame.

In any discussion of an atomic time scale it is essential that there be understanding as to the basic terms of reference and terminology. Such terms include *accuracy*, *stability*, *precision*, and *reproducibility*. The reader is referred to Chapter 8 for comprehensive definitions of these expressions.

9.3. CLOCK MODELING

In this section we will assume that we have a perfect clock reference denoted by t. In theory when we assume certain clock models and simulate data based on those models the above assumption is totally valid. In practice we make an effort to approach its total validity.

As it is often much easier to deal with residuals, we will refer to a clock's time difference from the ideal (or its estimation of the ideal in practice). This section describes deterministic properties, random perturbations of a clock's time, and clock noise characterization and data simulation.

9.3.1. Deterministic Properties

Deterministic properties indicate frequency offset or time drift, frequency or time jumps and other kinds of clock aging. We consider various aspects of such factors below:

a. *Accuracy and precision of synchronization.* The apparent time of the ith clock is:

$$t_i = N_i \tau_i + T_i(t_0) + t_0, \tag{9.2}$$

where $T_i(t_0)$ denotes the difference from ideal time of its reading at t_0, N_i is the number of its periods that have transpired since t_0 and τ_i is the period of the ith clock (the number of seconds per cycle). The accuracy of its reading at a particular time t is given by the actual value of $T_i(t) = t_i - t$ (how well it is synchronized); whereas, the time precision is given by the uncertainties in how well the value of $T_i(t)$ is known. If the rate of the ith clock is correct ($\tau_i = \tau_I$), then the accuracy and precision at any time t are limited only by the accuracy and precision at the origin, $T_i(t_0)$—essentially unrealizable in practice.

b. *Accuracy and precision of clock rate or frequency.* If the frequency of the ith clock is not correct ($\tau_i \neq \tau_I$) but is constant then the reading of the clock will diverge from the ideal reference. The residual time difference $T_i(t) = t_i - t$ will be given by:

$$T_i(t)=N_i\tau_i-N_I\tau_I+T_i(t_0), \tag{9.3}$$

which may be rewritten as

$$T_i(t)=R_i\times[t-t_0]+T_i(t_0). \tag{9.4}$$

R_i denotes the relative rate offset of the ith clock; e.g., a particular clock may differ in frequency such that it runs fast $+8.64$ nanoseconds per day (86400s). This relative rate offset is equivalent to the fractional frequency which we will denote by y_i (in this case $y_i=10^{-13}$), where

$$y_i=\frac{\nu_i-\nu_I}{\nu_I}. \tag{9.5}$$

The accuracy of the fractional frequency of the ith clock is given by an estimate of y_i; whereas, the precision of its fractional frequency is given by the uncertainties in how well the value of y_i is known.

c. *Frequency drift of a clock.* Essentially, all quartz crystal oscillators and rubidium gas cell frequency standards exhibit nonzero linear frequency drift [14]. Recently there has been documentation of the same phenomena in some commercial cesium beam frequency standards [2], and we will give additional documentation later in this chapter. Obviously, in this case the rate does not remain constant and we must modify eq (9.4) to give the following model:

$$T_i(t)=\tfrac{1}{2}D_i\times[t-t_0]^2+R_i(t_0)\times[t-t_0]+T_i(t_0), \tag{9.6}$$

where D_i is the fractional frequency drift per unit time and is here assumed constant. Note, the rate is now specified at a particular date.

d. *Other deterministic perturbations.* Occasionally—due to some perturbation either external or internal—a clock will manifest a different drift rate, a step change in its rate, or a step in its time. Mechanisms and/or methods to detect these kinds of perturbations should therefore be employed in order to affect a uniform time scale. In Section 9.4.3 we will discuss some possible methods of detection.

9.3.2. Random Perturbation of a Clock's Time

After the deterministic processes are properly assessed and accounted for in a clock, there will still remain a time deviation from this deterministic model, e.g., see eq (9.6). These deviations or fluctuations will be classified as random. Later, a causal relationship may be found which will account for some part of these deviations, but until such time, it is useful to use statistical techniques to categorize these random processes. This section describes the random (nondeterministic) fluctuations in a time scale; it gives statistical tools for modeling frequency stability and time dispersion.

a. *Statistical Models for Frequency Stability.* As a general model of clock behavior, let $x(t)$ denote these random deviations, and we will add this term to (9.6) to give:

$$T_i(t)=\tfrac{1}{2}D_i\times[t-t_0]^2+R_i(t_0)\times[t-t_0]+T_i(t_0)+x_i(t). \tag{9.7}$$

Dr. James A. Barnes has conducted some tests on the distribution of $x(t)$ for cesium and quartz clocks, and in those cases it was found to be normal. If we can assume normality, and know the spectral density or the auto-correlation function for $x(t)$, we would then have a complete statistical description of this process. Most high performance clocks can be well modeled statistically with a power law spectral density; i.e.

$$S_y(f)=h_\alpha|f|^\alpha, \tag{9.8}$$

where f is the Fourier frequency, h_α is the amplitude of the spectral density for a particular power law α, and $S_y(f)$ denotes the one-sided spectral density of the fractional frequency fluctuations y, and y is normally distributed [15, 16]. It should be noted that equation (9.5) for y_i is a fractional frequency offset, where y denotes flucations. The two designations differ only by a delta function at zero Fourier frequency, which is uninteresting and has caused little or no confusion in the past.

Typically, clock data (time or phase points) are taken in the time-domain; and time-domain stability measures involving auto-correlation functions are useful. In particular, we have employed the Allan variance [15, 16]:

$$\langle\sigma_y^2(N,T,\tau,f_h)\rangle=\left\langle\frac{1}{N-1}\left[\sum_{j=1}^{N}y_{(j)}^2-\frac{1}{N}\left(\sum_{j=1}^{N}y(j)\right)^2\right]\right\rangle, \tag{9.9}$$

where $y(j)$ is the jth data point in a sequence of fractional frequency data samples each of which is averaged over a nominal time τ and at a sampling rate $1/T$; f_h is the high frequency cutoff (the measurement system's effective bandwidth), and the angle brackets denote infinite time average. Two convenient relations may quickly be shown by setting $N=2$ and $T=\tau$ in eq (9.9), and we then have,

$$\langle\sigma_y^2(2,\tau,\tau,f_h)\rangle\equiv\sigma_y^2(\tau)=\left\langle\frac{[y(j+1)-y(j)]^2}{2}\right\rangle, \tag{9.10}$$

or

$$\sigma_y^2(\tau)=\frac{1}{2\tau^2}\,[4U_x(\tau)-U_x(2\tau)], \qquad (9.11)$$

where

$$U_x(\tau)=\langle[x(t)-x(t+\tau)]^2\rangle. \qquad (9.12)$$

Note, that f_h is not explicitly indicated as a variable in eqs (9.10)–(9.12), because typically it is held constant for a given experiment; none-the-less, its value should be stated. Though the data may have been taken with $T\neq\tau$, it is often possible to convert eq (9.9) for any N, T, and τ to that given by eq (9.10) (see ref. [17], also chap. 8). This particular autocorrelation function $U_x(\tau)$ is useful in dealing with some of the low frequency divergent power law spectral densities; viz., flicker noise frequency modulation [18]. Using the appropriate Fourier transform relationships one can write the following:

$$\sigma_y^2(\tau)=\frac{2h_\alpha}{(\pi\tau)^2}\int_0^{f_h} df\, f^{\alpha-2}\sin^4(\pi\tau f), \qquad (9.13)$$

where the spectral density is given by eq (9.8) and α is greater than -3[16]. Equation (9.10) is a very simple time-domain calculation (the infinite time average is normally well approximated with a few hundred data points in a sequence). Equation (9.13) gives a convenient relationship between these frequency-domain and time-domain measures of stability. For further details see Chapter 8 or reference [16].

b. *Statistical Models for Time Dispersion.* Typically the greatest contribution to the time dispersion in a clock comes from the first two terms on the right of eq (9.7); i.e., frequency drift and frequency offset. If, however, these can be measured to first order and accounted for, then the last term $x(t)$ becomes the primary contribution to the time dispersion. In practice, of course, the first three terms of eq (9.7) are observed only in the presence of the noise, $x(t)$. If the spectral character of the noise is known, then in principle one could design an optimum filter in some sense to best examine the signal for frequency drift, frequency offset, and time residual through the noise. In contrast, improper use of the time data from a clock may result in much larger errors than from optimal usage of the statistical data. In such sense, this is a fairly classical problem in signal detection in the presence of noise.

We will illustrate the above using the simple case of white noise frequency modulation $[S_y(f)=h_0]$. For this case eq (9.12) takes the form $U_x(\tau)=\frac{h_0}{2}\tau$, and therefore:

$$\sigma_y^2(\tau)=\frac{h_0}{2\tau}. \qquad (9.14)$$

It is easily shown that for this kind of noise the optimum estimate of a clock's rate (i.e., the signal) over a calibration interval τ_c is obtained by simply using the time of the clock at the beginning and end of the interval; i.e.,

$$\hat{R}_i\left(t-\frac{\tau_c}{2}\right)=\frac{T_i(t)-T_i(t-\tau_c)}{\tau_c}, \qquad (9.15)$$

where the "ˆ" over the R_i indicates an estimate. The mean-square time error realized by using this equation is given by

$$\langle\epsilon_i^2(\tau)\rangle=\langle[\hat{T}_i(t+\tau)-T_i(t+\tau)]^2\rangle, \qquad (9.16)$$

where $\hat{T}_i(t+\tau)$ denotes the time residual calculated from the rate given by eq (9.15), linear prediction rather than the exact rate. Equation (9.16), after substitutions and time averages are taken, becomes

$$\langle\epsilon_i^2(\tau)\rangle=\frac{1}{\tau_c^2}\,[\tau_c(\tau_c+\tau)U_{x_i}(\tau)+\tau(\tau_c+\tau)U_{x_i}(\tau_c)$$
$$-\tau_c\tau U_{x_i}(\tau_c+\tau)]. \qquad (9.17)$$

After substituting the above value of $U_x(\tau)$ for white noise frequency modulation (FM), eq (9.17) becomes

$$\langle\epsilon_i^2(\tau)\rangle=[\sigma_{y_i}^2(\tau_c)]\,[\tau_c+\tau]\tau. \qquad (9.18)$$

Let us examine three cases for eq (9.18); viz.,

$$\langle\epsilon_i^2(\tau)\rangle\cong\tau_c\sigma_{y_i}^2(\tau_c)\tau, \quad \tau<<\tau_c. \qquad (9.18a)$$

$$\langle\epsilon_i^2(\tau)\rangle=2\sigma_{y_i}^2(\tau_c)\tau_c^2, \quad \tau=\tau_c. \qquad (9.18b)$$

$$\langle\epsilon_i^2(\tau)\rangle\cong\sigma_{y_i}^2(\tau_c)\tau^2, \quad \tau>>\tau_c. \qquad (9.18c)$$

Note two things: first, the squared error is twice as large if the calibration interval is only equal to τ, the prediction interval, (eq (9.18b)) as compared to having a very long calibration interval relative to the size of τ (eq (9.18a)); second, the squared error disperses as τ^2 if the calibration interval is much shorter than τ (eq (9.18c)), which points out that the dispersion is not due to the noise after the calibration, but rather to the noise during the calibration causing an

TABLE 9.1. Frequency-Domain and Time-Domain Characteristics of Various Power Laws

Power Laws	Characterization			
	$S_y(f)$	$\sigma_y^2(\tau)$	$U_x(\tau)$	$\langle \epsilon^2_{\text{opt}}(\tau) \rangle$
White Noise PM	$h_2 f^2$	$\frac{3h_2 f_h}{(2\pi)^2 \tau^2}$	$\frac{2h_2 f_h}{(2\pi)^2}$	$\sim$ Constant
Flicker Noise PM	$h_1 f$	$\frac{h_1}{(2\pi)^2\tau^2}[9/2+3\ln(2\pi f_h \tau)-\ln(2)]$	$\frac{2h_1}{(2\pi)^2}[9/2+3\ln(2\pi f_h \tau)-\ln(2)]$	$\sim$ Constant + $\ln(\tau)$
Random Walk PM or White Noise FM	h_0	$\frac{h_0}{2\tau}$	$\frac{h_0}{2}\tau$	$\sim \tau$
Flicker Noise FM	$h_{-1} f^{-1}$	$2h_{-1}\ln(2)$	$\lim_{\alpha \to -1} \frac{2h_{-1}\ln(2)\tau^{-\alpha+1}}{2-2^{-\alpha}}$	$\sim \tau^2$
Random Walk FM	$h_{-2} f^{-2}$	$\frac{(2\pi)^2}{6} h_{-2}\tau$	$-\frac{(2\pi)^2}{12} h_{-2}\tau^3$	$\sim \tau^3$

PM ≡ Phase Modulation; FM ≡ Frequency Modulation; for White Noise PM and Flicker Noise PM we assume $\tau f_h \gg 1$.

error in determining the clock's rate, $\hat{R}_i\left(t-\frac{\tau_c}{2}\right)$, over too short a calibration interval. Data can be and often are misused in this manner—yielding a much greater time dispersion than would be necessary.

It is useful to tabulate some of the common kinds of power law noise processes which perturb the outputs of the high performance clocks currently available. In table 9.1 these power laws are listed along with their frequency-domain and time-domain characterization. The equation for $U_x(\tau)$ and the basic behavior for the mean square time dispersion after optimal data usage are also listed in table 9.1. It should be clearly noted that the time dispersion is dependent upon how the data are utilized, and in some cases it is highly dependent, as illustrated above for white noise FM, where for too short a calibration interval the mean square time dispersion goes as τ^2 rather than as τ.

White noise PM and Flicker noise PM are typically the predominant noise processes for values of τ less than 1 second; e.g., in quartz crystal oscillators and hydrogen maser frequency standards. For longer than one second, the predominant noise processes are usually white noise FM, flicker noise FM, and in some cases random walk FM. One or more of these last three noise processes are applicable in many frequency standards; e.g., cesium beam, hydrogen maser, rubidium gas cell, quartz crystal oscillator, and probably any oscillator stabilized by a passive frequency resonance as in the methane stabilized helium-neon laser.

Figure 9.1, showing some time-domain stability characteristics of various frequency standards measured at NBS, illustrates most of the above kinds of noise processes. As a stability plot is analyzed, it is useful to note and it may be shown [15, 16, 19] that if: $\sigma_y^2(\tau) \sim \tau^\mu$, then

$$\mu = \begin{cases} -2, \text{ for } \alpha \geq 1 \text{ and } |\tau f_h| \gg 1; \\ -\alpha - 1, \text{ for } -3 > \alpha \leq 1. \end{cases} \quad (9.19)$$

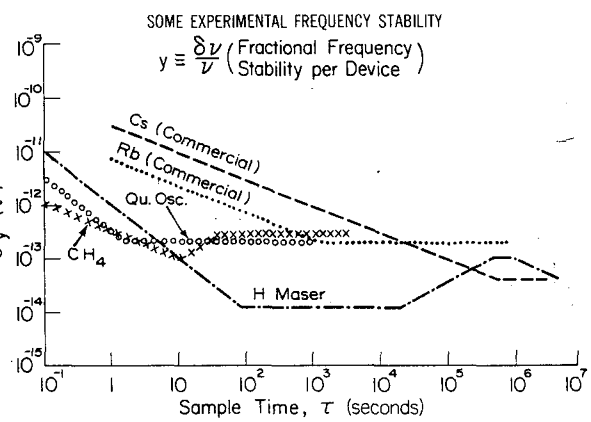

FIGURE 9.1. Fractional frequency stability of several types of oscillators as a function of sample time.

Annex 9.A considers an optimum filter (optimum in the sense of giving a minimum squared error of time prediction $\langle\epsilon^2_{\rm opt}(\tau)\rangle$) in detail for each of the above kinds of noise processes. If a process can be reduced to that of white noise through linear transformation, then the simple mean of the transformation gives the optimum predictor for this white noise process. By taking the inverse transform, one can generate the optimum filtered output for minimum squared error time prediction for the process itself [20].

At this point we will illustrate a very simple recursive relationship which yields near optimum time prediction for some important practical instances. Consider specifically the noise process for $\alpha=-1$; this is a noise type commonly observed in long term in high performance clocks. In this case $\sigma^2_y(\tau)$ is independent of τ. (We will not treat the case where $\alpha=1$ as it has much less relevance for timekeeping; however, the same principles apply to both flicker noise FM and flicker noise PM).

Consider a sequence of measurements of the fractional frequency on the ith clock, $y_i(j)$, where the j denotes the particular member of the sequence as in eq (9.9). An exponential weighting of the past measurements to yield a near optimum estimate for the current frequency of an $\alpha=-1$ noise process is given approximately by the following recursion relationship [21];

$$\hat{y}_i(j)=\frac{1}{m+1}\left[y_i(j)+m\hat{y}_i(j-1)\right]. \qquad (9.20)$$

The time constant of the exponential weighting filter is given by m. The general expression for the squared error of prediction is given by:

$$\begin{aligned}\langle\epsilon^2(\tau)\rangle=&\,U_x(\tau)+\frac{U_x(\tau)}{m^2}\sum_{k=1}^{\infty}\left(\frac{m}{m+1}\right)^{2k}\\ &-\frac{2}{m}\sum_{k<l}^{\infty}\left(\frac{m}{m+1}\right)^{k}\left\{\frac{1}{2}\left[U_x\Big((k+1)\tau\Big)\right.\right.\\ &\left.\left.+U_x\Big((k-1)\tau\Big)\right]-U_x(k\tau)\right\}\\ &+\frac{2}{m^2}\sum_{kl}^{\infty}\left(\frac{m}{m+1}\right)^{k}\left(\frac{m}{m+1}\right)^{l}\\ &\left\{\frac{1}{2}\left[U_x\Big((l-k-1)\tau\Big)+U_x\Big((l-k+1)\tau\Big)\right]\right.\\ &\left.-U_x((l-k))\right\}. \end{aligned} \qquad (9.21)$$

For a given noise process and a given prediction interval τ, there exists a value of m which will give a minimum squared error for eq (9.21).

Substituting the equation for $U_x(\tau)$ for flicker noise FM ($\alpha=-1$) from table 9.1 into eq (9.21) and choosing the optimum value of m ($m=0.6$ for $\alpha=-1$) yields the result that the root-mean-squared error given by eq (9.21) is only a factor of 1.13 times larger than that for an optimum prediction routine. The residual time calculation is based on the operationally very simple recursion relationship for the rate given in eq (9.20).

9.3.3. Clock Noise Characterization and Data Simulation

Once the deterministic and random properties of a clock have been estimated, then it is possible to simulate a clock's behavior. The simulation of the deterministic part is, of course, very straightforward using, step, ramp, and quadratic functions. The random portion, characterized by power law spectral densities ($S_y(f)=h_\alpha f^\alpha$), may be simulated using Gaussian white noise generators and the white noise to flicker noise filters, including integrals of the same [22]. The deterministic part of a clock's behavior is usually well modeled by eq (9.6). Quite commonly we find that the random part, $x(t)$, is well modeled by the following equation (see fig. 9.1):

$$S_y(f)=h_0+h_{-1}f^{-1}, \qquad (9.22)$$

or in the time-domain:

$$\sigma^2_y(\tau)=\frac{h_0}{2}\tau^{-1}+h_{-1}2\ln(2), \qquad (9.23)$$

where

$$y(t)=\frac{dx(t)}{dt}. \qquad (9.23a)$$

Assuming the statistical model of eqs (9.22) and (9.23), we will calculate the mean-squared time error for an interval τ for the two useful time calculation techniques discussed above, viz., linear prediction and exponential prediction. First, consider the calibration interval τ_c with the residual rate of a clock being calculated using eq (9.15). Then using the appropriate equation for $U_x(\tau)$ from table 9.1 and substituting these into eq (9.17) gives:

$$\begin{aligned}\langle\epsilon^2(\tau)\rangle=&\,\frac{h_0}{2\tau_c}(\tau_c+\tau)\cdot\tau\\ &+h_{-1}\tau^2\left[\frac{\tau}{\tau_c}\left(\frac{\tau_c}{\tau}+1\right)^2\ln\left(\frac{\tau_c}{\tau}+1\right)-\left(\frac{\tau_c}{\tau}+1\right)\ln\left(\frac{\tau_c}{\tau}\right)\right].\end{aligned} \qquad (9.24)$$

For a given set of noise levels h_0 and h_1 and for a particular prediction interval τ, there is a value of τ_c that will give a minimum for eq (9.24). Let us examine the following special cases:

$$\langle \epsilon^2(\tau) \rangle \simeq \frac{h_0}{2}\tau + h_{-1}\left(1+\ln\left(\frac{\tau_c}{\tau}\right)\right)\tau^2, \qquad \tau \ll \tau_c. \tag{9.24a}$$

$$\langle \epsilon^2(\tau) \rangle = h_0\tau + 4h_{-1}\ln(2)\cdot\tau^2, \qquad \tau = \tau_c. \tag{9.24b}$$

$$\langle \epsilon^2(\tau) \rangle \simeq \frac{h_0}{2\tau_c}\tau^2 + h_{-1}\left(1-\ln\left(\frac{\tau_c}{\tau}\right)\right)\tau^2, \qquad \tau \gg \tau_c. \tag{9.24c}$$

Note in eq (9.24a) that as the calibration interval grows even longer, the error deteriorates only very slowly for the flicker noise FM contribution (second term on right), and the error approaches a minimum (optimum) for the white noise FM contribution (first term on right). Note, in eq (9.24c) that the mean squared time error goes as τ^2 for both terms, and which one predominates depends not only on the intensity of each kind of noise (h_0 and h_{-1}), but also on the calibration interval (τ_c).

In figure 9.2 we have plotted the root-mean-square time error given by eq (9.24) for two atomic clocks with different levels of noise. The time fluctuations are assumed to follow the model in eqs (9.22) or (9.23), and the values for the noise intensity were chosen to nominally cover most of the commercial cesium beam clocks presently being used in the field. This illustration assumes that the deterministic terms in eq (9.7) are accounted for, and the time dispersion is due primarily to $x(t)$ at the noise intensities indicated.

Second, for exponential prediction, consider the mean-square time error in calculating time based on eq (9.20) and assuming the same statistical model; viz., that given in eqs (9.22) or (9.23). Again taking the appropriate value of $U_x(\tau)$ from table 9.1 and substituting into eq (9.21) gives:

$$\langle \epsilon^2(\tau) \rangle = \frac{h_0(m+1)}{2m+1}\tau + \frac{h_{-1}\tau^2}{m^2(2m+1)}\sum_{k=1}^{\infty}\left(\frac{m}{m+1}\right)^k k^2\ln(k). \tag{9.25}$$

It is again worth noting that the value of m (the exponential frequency-weighting time constant) which gives a minimum for eq (9.25) will depend upon the level of white noise FM (h_0) and flicker noise FM (h_{-1}). For convenience we have tabulated in table 9B.1 (ann. 9.B) some useful values of the infinite sum in the second term on the right of eq (9.25). Note, as m becomes large, the first term on the right of eq (9.25) approaches the optimum. Assuming that $\frac{\tau_c}{\tau} = m$ in eq (9.24), it can be shown that eq (9.25) always gives at least a slightly smaller value for the error than does eq (9.24). For $m \ll 1$, both terms on the right of eq (9.25) become significantly less than in eq (9.24); and specifically, comparison of the terms due to white noise FM show that eq (9.24) gives a squared error which is 1/(2m) larger than in eq (9.25).

A comparison of the squared error due to the flicker noise FM terms (second term on right) in eqs (9.24) and (9.25) is shown in figure 9.3.

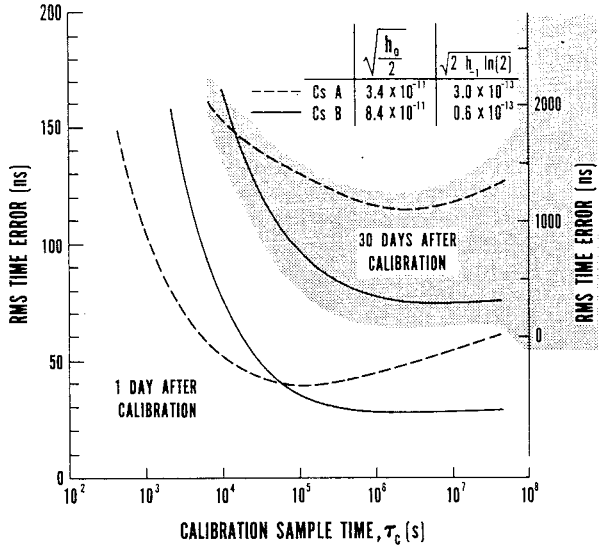

FIGURE 9.2. Root-mean-square time error of two cesium clocks as a function of calibration sample time.

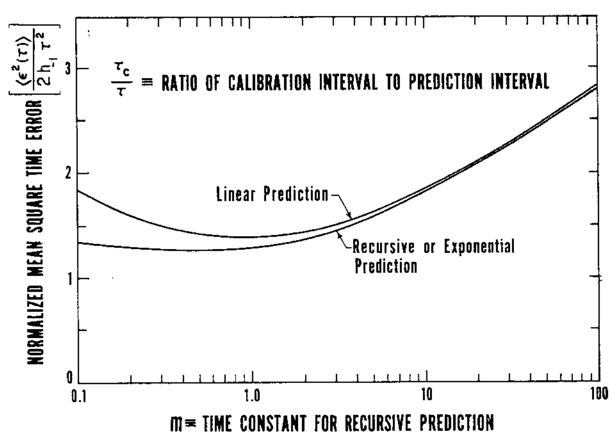

FIGURE 9.3. Comparison of mean-square time error of linear and exponential predictors for flicker noise FM.

Note, the minimums occur at $m=1$ for the term from eq (9.24) and for $m=0.6$ for the term from eq (9.25).

Although there may be any number of time prediction algorithms their efficiency and accuracy may be determined by applying certain tests; e.g., by processing simulated data; comparing with optimum prediction for the models assumed; and making sure that the algorithm is not highly model dependent. (In other words, the kinds of noise and the levels of noise don't have to be critically determined.) The two prediction algorithms whose frequencies are given by eqs (9.15) (linear) and (9.20) (exponential) pass these tests as long as m is approximately equal to or larger than its optimum value. They are both very easy to implement; eq (9.20) is easier to employ and is more nearly optimum. We have developed a first order time scale algorithm based in part on eq (9.20).

9.4. THE AT(NBS) TIME SCALE SYSTEM

Figure 9.4 is a functional diagram of the NBS Atomic Time Scale system (AT(NBS)). The theory of operation is as follows: Block A represents a device which will accurately produce the SI second or a known fraction thereof. Block B denotes a precise measurement of the frequency (ν_i) or rate (R_i) and hence of the period (τ_i) of each of n clocks. Block C represents a set of n independent clocks with fail-safe power supplies, where n is large enough to do individual clock characterization and to provide sufficient redundancy to guarantee that clocks will always be running. Each clock serves as an independent memory of ν_0 or τ_0; and together they evaluate each other. Block D denotes a precise and accurate measurement of the time differences between the clocks and indicates the mechanism for this evaluation. The time differences and the rate information are optimally used in a time scale algorithm to produce a time as shown in block E. This time is as near synchronous as possible with ideal time as discussed earlier. The time derived from the primary frequency standard, had it been running continuously at its correct (evaluated) rate, is also an approach to ideal time, but in practice there are two reasons that make this difficult to achieve. We will discuss these reasons and the functions indicated by each block in figure 9.4 in detail.

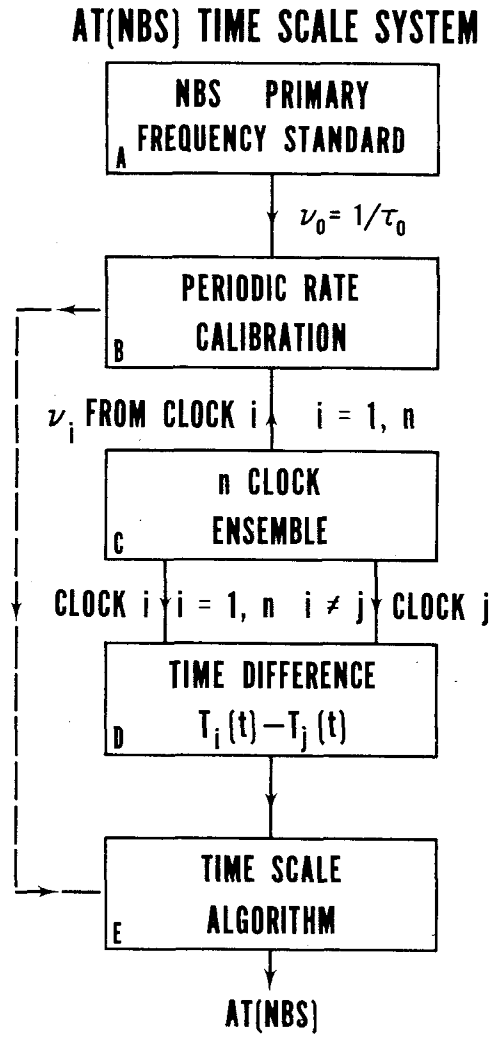

FIGURE 9.4. Block diagram of the AT(NBS) time scale system.

9.4.1. The NBS Primary Frequency Standard and Measurement System

Historically, the primary frequency standard at NBS has been a very elaborate laboratory cesium beam device which cannot reasonably be operated continuously. In fact the decision for non-continuous operation was made so that we could properly evaluate all known parameters which may affect the accuracy realizing ν_0. The last full evaluation and calibration of the NBS–III primary frequency standard (May 1969 [23]) indicated an accuracy of 5×10^{-13} (1σ). Two new state-of-the-art primary standards have been constructed at NBS, viz., NBS–X4 and NBS–5. A preliminary evaluation of NBS–5 has been made in late 1972 and early 1973, and the estimated frequency of NBS–III via the AT(NBS) clock ensemble is within 1 part in 10^{13} (1σ confidence of 1 part in 10^{12}) with respect to the best estimate of frequency given by NBS–5.

NBS–X4 is still to be evaluated, and the final evaluation on NBS–5 is proceeding at the time of this writing. When the primary frequency standard is operating, a 5-MHz signal, ν_0', is synthesized from this standard and compared with a 5-MHz source from each of the clocks in the ensemble. The prime on ν_0 in this text denotes a known frequency offset between the synthezised signal and the best estimate of a 5-MHz signal based on the primary cesium beam standard. The comparison is performed in a low noise double-balanced Schottky barrier diode mixer. The mixer provides the difference (beat) frequency ($\Delta\nu_i = \nu_i - \nu_0'$) between the primary standard and the ith clock. The equation for the precision of such a measurement system is given by:

$$\sigma_y(\tau) = \frac{\delta\tau_b}{\tau}\frac{\Delta\nu}{\nu}, \qquad (9.26)$$

where $\delta\tau_b$ is the precision with which the beat period ($\tau_b = 1/\Delta\nu$) is known, and τ is the time interval over which the beat signal is sampled; i.e., sample time.

The AT(NBS) rate measurement system is called a chronograph; the chronograph data, occurence times of the zero crossings of the beat frequency ($\Delta\nu$) on an arbitrary scale, are measured to a precision of 10μs. In a typical situation we may have a value of τ_b of 100s, which gives $\sigma_y(\tau) \simeq 2\times 10^{-14}/\tau$ ($T \geqslant 100$s).

A chronograph system has the theoretical potential of an extremely precise time interval device: e.g., in the above example one could measure time (phase) fluctuations between two clocks to a precision of 20 femtoseconds (20×10^{-15}s) since, from eq (9.25), we may derive:

$$\delta(\Delta t) = \frac{\delta\tau_b}{\tau_b \nu}, \qquad (9.27)$$

where $\delta(\Delta t)$ denotes the uncertainty of the change in the time (phase) difference measurement.

In practice dc drifts and component delay variations will prohibit achieving the precision given by eq (9.27). Some tests have been conducted on the chronograph which have indicated that its contribution to the instabilities of a measurement are usually negligible—consistent with eq (9.26).

9.4.2. The Clock Ensemble and Time Difference System

There are currently eight commercial cesium beam frequency standards in the AT(NBS) clock ensemble. Six of these units are located at the NBS/Boulder, CO laboratories in an environmental chamber which is controlled in temperature to better than 0.1 degree C at a nominal ambient temperature of about 23 degrees C. Each of these six units is also electrically isolated and shock mounted with an independent fail-safe power supply. The whole system is backed up with an emergency generator power source. Each of these six standards drives two frequency dividers—one from 5 MHz to 1 pulse per second (pps) and the other from 100 kHz to 1 pps. The latter divider is a redundant backup. The 5-MHz dividers are classically called "window" dividers and have pulses which are stable to subnanosecond with 6 ns rise times. The other two atomic frequency standards are at the WWV radio station near Ft. Collins, CO in a shielded and temperature controlled room. They are similarly followed by dividers from 5 MHz to 1 pps and the occurrence times of these clocks are communicated to Boulder, CO via the TV line-10 time transfer system [7, 9].

Time differences are measured to an accuracy of 2 ns and a precision of 0.5 ns with a commercial time interval counter. The time differences between one specific clock and all the others are measured automatically at the same time each day; an on-line computer is available for processing and diagnosing the data. In practice the 1 pps signal from clock i starts the counter and at a time τ later the 1 pps signal from clock j stops the counter; this gives:

$$T_i(t) - T_j(t+\tau) = T_i(t) - T_j(t) + \tau\bar{y}_j^{\tau}(t). \qquad (9.28)$$

The third term on the right of eq (9.28), $\tau\bar{y}_j^{\tau}(t)$, denotes τ times the average fractional frequency offset of the jth clock over the interval from t to $t+\tau$. This term is usually negligible since $\tau < 1$ s and $\bar{y}^{\tau}(t) < 10^{-11}$. However, caution should be used if one has a clock with a large fractional frequency offset, since this third term usually is ignored in practice.

9.4.3. Atomic Time Scale Algorithms

In theory one could perform a complete characterization of both the random and the deterministic properties of each member of a set of clocks in a given ensemble. After such a characterization, favorable processors could be applied to the data to yield a theoretically optimum time scale. The obvious problem with this approach is that optimum time cannot be known during any part of the characterization interval until after all the data have been processed which may require weeks, months or years.

Another problem is—even though eqs (9.7) and (9.8) are believed to be good models for high performance clock data—occassionally a change may occur in one of the coefficients, i.e., D_i, $R_i(t_0)$, $T_i(t_0)$, h_{0_i}, or h_{-1_i}, (e.g., the cesium getters saturate in the clock causing h_0 to increase). The changes may occur discretely or gradually. Methods of detection should, therefore, be incorporated to sense such changes. One has the additional theoretical complication that a gradual change in $T_i(t_0)$, starting at some date t, cannot be separated from a discrete change in $R_i(t_0)$, at that same date; a similar situation exists for $R_i(t_0)$ and D_i respectively. In practice, however, it is immaterial as to which assignment is made, and one is prone to choose the most convenient.

In light of the above we take the following approach for the AT(NBS) scale. With a very small sacrifice in accuracy of dating we maintain an online clock which constantly predicts the date that would be given by the clock ensemble (first order processed), and which is updated after each periodic (~ once per day) ensemble processing. This first order time scale algorithm has dynamic sensing qualities; i.e., after the data are processed through a near optimum filtering routine, the algorithm senses and compensates for any changes in $R_i(t_0)$, $T_i(t_0)$, and in the noise intensity. After a sufficient amount of data has been collected (over months or years), a second order time scale algorithm is applied to the data to give a best estimate of dating from the ensemble and primary frequency standard, and to update the coefficients for the first order algorithm. We will discuss the first order algorithm in some detail, but only some aspects of the second order algorithm as it is still being developed and tested.

a. *Clock Characterization.* In order to best estimate the deterministic coefficients (signal) for eq (9.7), we first estimate the noise characteristics so that suitable data filtering can be employed to yield near optimum signal to noise. The noise characteristics of the ith clock can be estimated by comparing it with a clock whose noise is less that its own noise over the stability region of interest (the values of τ). An example of this is shown in figure 9.5, where an NBS clock is compared with a Hydrogen Maser. It is highly desirable to use the "best" clock in the ensemble, and hence a different technique needs to be employed to estimate its stability. If the three clocks i, j, and k are independent, then we may use the following as an estimate of the ith clock's stability:

$$\sigma_i^2(\tau) = \tfrac{1}{2}\left[\left(\sigma_{ij}^2(\tau) - \sigma_{n_{ij}}^2(\tau)\right) + \left(\sigma_{ik}^2(\tau) - \sigma_{n_{ik}}^2(\tau)\right) - \left(\sigma_{jk}^2(t) - \sigma_{n_{jk}}^2(\tau)\right)\right], \qquad (9.29)$$

where $\sigma_{ij}^2(\tau)$, etc., are the measured stabilities between i and j, etc., and the $\sigma_n^2(\tau)$ are the measurement noise contributions respectively. If the measurement noise, $\sigma_n^2(\tau)$, is the same for each measurement pair, then eq (9.29) may be approximated by:

$$\sigma_i^2(\tau) = \tfrac{1}{2}\left[\sigma_{ij}^2(\tau + \sigma_{ik}^2(\tau) - \sigma_{jk}^2(\tau) - \sigma_n^2(\tau)\right]. \qquad (9.29a)$$

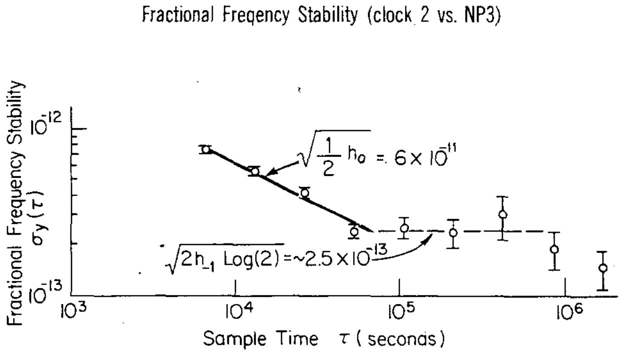

FIGURE 9.5. Hydrogen Maser (NP3), cesium beam (clock 2) comparison, showing the fractional frequency stability as a function of sample time.

Of course, $\sigma_n^2(\tau)$ needs to be measured as a separate experiment or calculated from known pertinent parameters.

Given n clocks in an ensemble there will be $(n-1)!/(2!(n-3)!)$ different, but not all independent, estimates of $\sigma_i^2(\tau)$. By taking an appropriate weighted combination—recognizing that some estimates will have much better confidences than others—we get a best estimate along with the confidence of the estimate for the stability of each clock in the ensemble.

During the winter of 1969 and 1970 we had the advantage of using the National Aeronautics and Space Administration's (NASA) NP3 hydrogen maser, for study of the above method of clock stability estimation [24]. Figures 9.6 and 9.7 are $\sigma_y(\tau)$ versus τ plots of NBS clocks 1 and 3, which show the estimated stabilities. After the stabilities of each clock were estimated, an approximation of the ensemble stability can be made by taking the appropriate weighted combination of the stability plots for each clock. These weighting factors are discussed below. Figure 9.8 shows such a stability plot for the NBS clock ensemble. By taking the stability estimate for the clock ensemble and combining it with the stability estimate for the NP3 hydrogen maser, we compared the estimated stability of the combination with that which was actually measured. Figure 9.9 shows the excellent agreement of this comparison.

b. *Calculation of Ensemble Time.* Assume that an optimal rate, $\hat{R}_i(t_0)$, calibration has occurred for each clock using the clock ensemble and/or

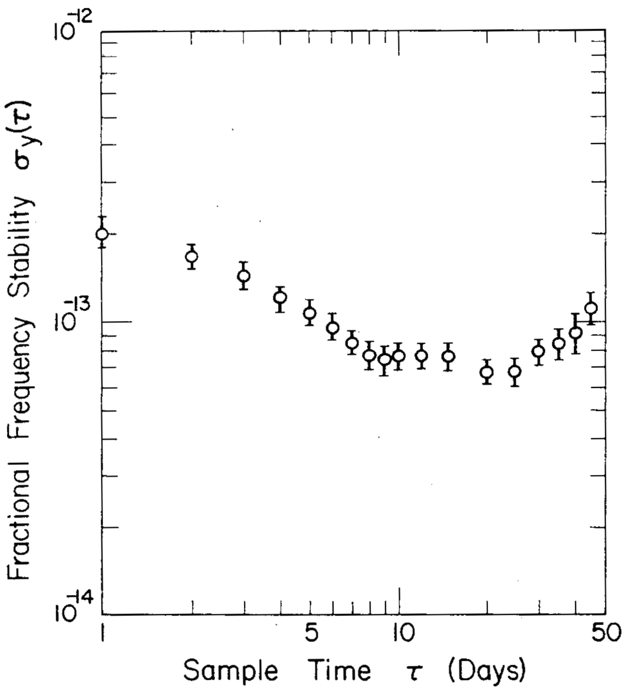

FIGURE 9.6. Fractional frequency stability of clock 1 estimated by comparison with all other clocks in the AT(NBS) clock ensemble.

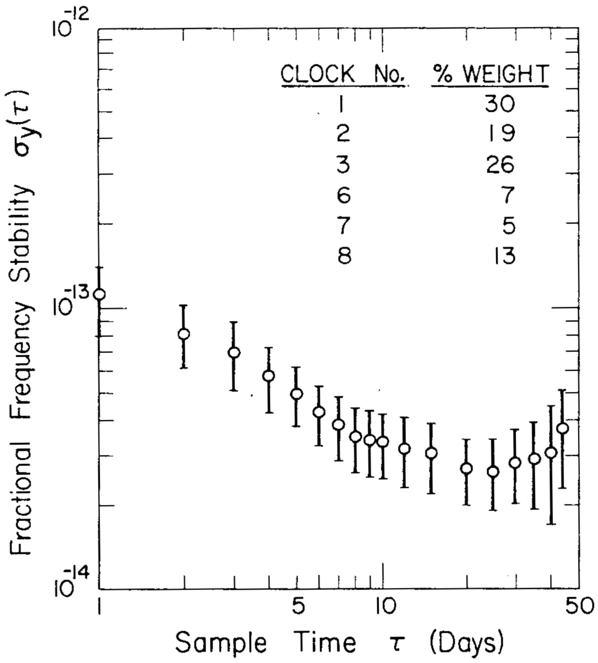

FIGURE 9.8. Fractional frequency stability of weighted 6 clock ensemble as determined from individual clock-estimates.

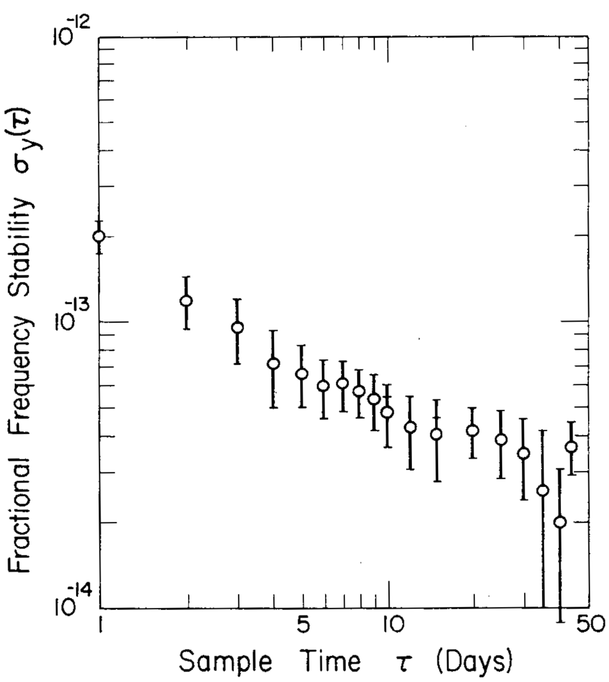

FIGURE 9.7. Fractional frequency stability of clock 3 estimated by comparison with all other clocks in the AT(NBS) clock ensemble.

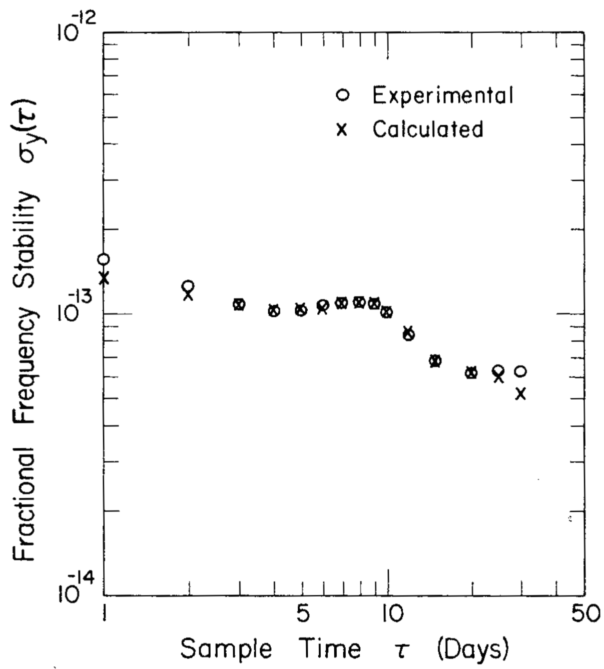

FIGURE 9.9. Hydrogen Maser (NP3) AT(NBS) clock ensemble (6 clocks) comparison showing experimental and calculated values of fractional frequency stability as a function of sample time.

the primary frequency standard, and assume further that the residual time difference between each clock and that time given by the ensemble at an agreed upon date t_0 is known, $T_i(t_0)$. We have had insufficient data to determine the D_i coefficients which require, ideally, fairly frequent and precise calibrations with an accurate primary standard whose intrinsic reproducibility is sufficiently good so that undue noise is not added to the stability of the clock ensemble. We know the D_i coefficients are usually small, but their quadratic time dispersion rate is probably not insignificant. We will assume that they are negligible for the first order algorithm, and give proper accounting for them in the second order algorithm. For the first order algorithm, we may then optimally predict the following residual time for clock i:

$$\hat{T}_i(t_0+\tau) = \hat{R}_i(t_0) \cdot \tau + T_i(t_0), \qquad (9.30)$$

neglecting second order terms.

Now we may combine the measured time difference data,

$$T_{ij}(t_0+\tau) \equiv T_i(t_0+\tau) - T_j(t_0+\tau),$$

with eq (9.30) for all the clocks to give:

$$T_j(t_0+\tau) = \sum_{i=1}^{n} w_i(\tau) \, [\hat{T}_i(t_0+\tau) - T_{i_j}(t_0+\tau)], \qquad (9.31)$$

where the w_i are appropriate weighting factors depending upon the quality of each clock. In order to have a minimum squared error of the weighted ensemble from ideal time, it may be shown that:

$$w_i(\tau) = \frac{\langle \epsilon_e^2(\tau) \rangle}{\langle \epsilon_i^2(\tau) \rangle} \qquad (9.32)$$

where $\langle \epsilon_e^2(\tau) \rangle$ is the estimated squared error of the ensemble and is given by

$$\langle \epsilon_e^2(\tau) \rangle = \left[\sum_{i=1}^{n} \frac{1}{\langle \epsilon_i^2(\tau) \rangle} \right]^{-1} \qquad (9.33)$$

From eq (9.33) one can see that *all* clocks *contribute positively* to the stability of the ensemble; i.e., a poor clock does not degrade the stability. Further, the stability of the clock ensemble is better than that of the most stable clock in the ensemble.

For reasons given in the previous section we currently use an exponential prediction routine in our first order algorithm. Therefore, we may calculate the mean-squared clock error using eq (9.25)—having estimated the noise characteristics using eq (9.29). We may also experimentally estimate the time dispersion of a clock if we have several time measurements with respect to the ensemble over equally spaced intervals. A bias is introduced when measuring a clock's time against the time of the ensemble of which it is a member since it would not, then, be statistically independent. An estimate of the unbiased error of the jth clock accumulated over the interval τ is given by:

$$|\epsilon_j(\tau)| = |\hat{T}_j(t_0+\tau) - T_j(t_0+\tau)| + \frac{0.8\langle \epsilon_e^2(\tau) \rangle}{\langle \epsilon_i^2(\tau) \rangle^{1/2}}, \qquad (9.34)$$

where the 0.8 arises from the assumption of a normal distribution of errors. The average square of eq (9.34) then gives an experimental estimate of eq (9.25). We have tested both methods of clock error estimation and obtained reasonable agreement between them.

Equation (9.31) gives the time of the jth clock with respect to ideal time, estimated by a procedure which minimizes the squared error; hence a working clock can be made to read "correctly" by adjusting its tick to be $T_j(t_0+\tau)$ later than that of the jth clock. Once the residual time of each clock has been calculated, we have in effect a new origin and in general we may replace t_0 by t in eqs (9.30) and (9.31). Generalizing these latter two equations for any time t, one has the advantage of both sensing and accounting for the previously mentioned dynamic changes in a clock's characterization.

All that is required to confidently sense rate changes or time jumps in a clock is a stable reference. Since the primary frequency standard is not continuously available, we simply use the clock ensemble, which in principle is better than the best clock in the ensemble, as our stable reference between calibrations. Equation (9.20) is used as a near optimal update for the rate $\hat{R}_i(t)$ of each clock. Figure 9.10 shows the rates (converted to fractional frequency, $y_i(t)$) for some of the clocks composing the AT(NBS) scale ensemble. These rates were determined with respect to the ensemble, not the primary standard. The ordinate was arbitrarily and conveniently chosen for each clock. Annex C gives a computer program written for a mini-computer, covering some aspects of our first order time scale algorithm which generated the rates shown in figure 9.10.

Whereas the weighting factors which give a minimum mean squared time error are given by eq (9.32), the time constant $m_i\tau$ (m as in eq (9.20)), coupled with w_i determine the weighting factors which give the near optimum frequency stability for the clock ensemble. A classic illustration of the above weightings is a quartz crystal clock as compared to a commercial cesium beam clock (see fig. 9.1). If the interval τ between time interval measurements were short (a few seconds) then a

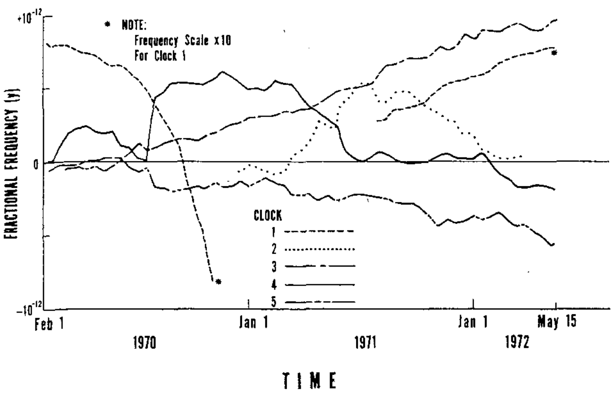

FIGURE 9.10. Relative fractional frequencies of five selected cesium clocks in the AT(NBS) scale as a function of time with the AT(NBS) scale used as the reference (note the arbitrary origin).

larger weight would be given to the quartz crystal clock (per eq (9.32)) than to the commercial cesium clock—yielding a very stable time scale in short-term. However, in long-term the superior frequency stability of the atomic clock shows an optimum value of m much larger than for the quartz crystal clock and yields a very stable time scale, as the composite rate would be determined by the commerical cesium beam clock.

For the above particular exponential time prediction algorithm, we have conducted tests to see how variations in the value of τ (the interval between time difference measurements) affects the overall stability characteristics of the ensemble. A small effect was observed; e.g., increasing τ by about a factor of 10 improved the long-term stability by a few percent. This is a direct indication that this particular algorithm is only near optimum. For practical applications, however, where clocks can only be characterized to confidences of a few percent, the ease of implementation of this exponential prediction routine has proved to be efficient and adequate for our first order time scale algorithm. The value of τ can be chosen pretty much for the convenience of data acquisition and analysis without any significant degradation of the time scale stability.

We currently measure the time differences between the clocks each day; such measurements are done automatically in our time scale automation system. Figure 9.11 is a plot of the accumulated time errors summed from the errors determined after each day's measurement for the more important clocks in the NBS clock ensemble. Currently for $\tau = 1$ day the experimental ensemble error given by eq (9.33) is about 5 ns. If we may assume that the ensemble stability plot shown in figure 9.8 is representative for longer values of τ, then we can use eq (9.25) to estimate the time dispersion of the clock ensemble for the 2 years and 3 months period shown for some of the individual clocks in figure 9.11. Assuming $m\tau$ for the clock ensemble is about 8 days, we derive a time dispersion for the clock ensemble of about 4μs, which appears conservative in comparison with the time dispersion of the individual clocks. Note, that this is the time dispersion due to the random error predominantly, and does not account for non-random dispersion such as may be caused by steps and drifts in frequency.

c. *Rate Accuracy of AT(NBS) Scale.* Ultimately the accuracy of the rate of an atomic time scale must depend upon the accuracy of one or more evaluable primary frequency (rate) standards. In our case a primary reference was NBS – III in the past, and is now NBS – 5 and NBS – X4. The primary function of a clock ensemble as regards rate accuracy is to serve as a memory of past calibrations. The better the clock ensemble stability the more nearly perfect the memory of the proper rate given by a primary standard from which the clock ensemble derives its accuracy.

In any given evaluation of a primary rate standard there are associated inaccuracies. Each of these inaccuracies arises from the uncertainty in knowing how much a particular parameter biases the frequency of the primary standard. It is convenient to categorize these bias uncertainties: first are those uncertainties $\sigma_{ruc}(l)$, which will be random and uncorrelated in their inaccuracy contribution from one evaluation to the next; second are those which are otherwise, $\sigma_b(l)$; these are typically constant for a given device. The overall accuracy may then be written for the lth evaluation:

$$\sigma(l) = [\sigma_{ruc}{}^2(l) + \sigma_b^2\,(l)]^{1/2}. \tag{9.35}$$

If we had a time scale with perfect memory the first inaccuracy contribution would average as the square root of the number of calibrations. In such a

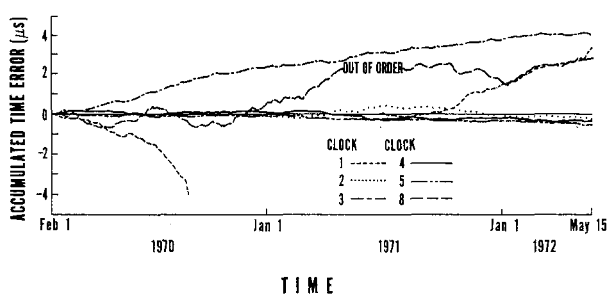

FIGURE 9.11. Long-term accumulated time errors of 6 selected cesium clocks in the AT(NBS) time scale.

case we could have a time scale with an accuracy of rate better than that from a single evaluation of the primary standard, given a sufficient number of calibrations. In practice, of course, there is some frequency dispersion in the time scale memory as well as the possibility of some deterministic drifts, $D_i \neq 0$ (see fig. 9.10). Assuming that the deterministic aspects such as D_i can be properly accounted for in the second order time scale algorithm and assuming that each calibration is optimally weighted, we may then write the following for the squared accuracy of the calibrated clock ensemble, $\sigma_e^2(l)$, after the lth calibration:

$$\sigma_e^2(l) = \sigma_b^2(l) + \left[\frac{1}{\sigma_{\text{ruc}}^2(l)} + \frac{1}{\langle \sigma_{y_e}^2(2,T,\tau,f_h)\rangle + \sigma_e^2(l-1) - \sigma_b^2(l-1)} \right]^{-1}, \tag{9.36}$$

where $\langle \sigma_{y_e}^2(2,T,\tau,f_h)\rangle$ is the clock ensemble frequency stability (see eq (9.9); (T is the time from the beginning of one calibration to the next, and τ is the calibration sample time). We have also implicitly assumed that the same primary standard is employed and that $\sigma_b^2(l)$ remains the same or improves with experience. If either or both of these assumptions are not true, one can improve on eq (9.36) through techniques to be published in the future.

From eq (9.36), we see that with an adequately stable clock ensemble, the accuracy of the clock ensemble approaches that of $\sigma_b(l)$. In the case of AT(NBS) the last calibration was May 1969, and $\sigma_e^2(l-1)$ was 5×10^{-13} [23]. Our first preliminary calibration of AT(NBS) with NBS-5 occurred over the period 13–22 January 1973 with preparatory measurements being made during November and December of 1972. This calibration measured AT(NBS) too high in rate by 6.9 parts in 10^{13}. The use of eq (9.36) and weighting of the two above mentioned calibrations inversely proportional to their accuracies indicated that we should decrease the rate of AT(NBS) by 4.5 parts in 10^{13}; this change was made at 0000 hours AT on 1 February 1973. Between 27 January 1973 and 6 April 1973 we performed four more preliminary calibrations, and again the application of eq (9.36) for each of these indicated that we should increase the rate of AT(NBS) by 5.1 parts in 10^{13}.

These experimental results give direct evidence for the second reason we believe it is not best to use a primary frequency standard as a clock; i.e., the biases which need to be taken into account in order to obtain an accurate frequency estimate are not always constant from one calibration to the next. Hence, if the clock ensemble is more stable than the changes in these biases, an appropriate filter may be applied to the calibration values so that both accuracy and stability are preserved for the atomic time scale. Incorporating this philosophy we increased the rate of AT(NBS) by 0.5 parts in 10^{13} – a frequency step which is the order of the time scales frequency instability – at 0000 hours AT, 1 May 1973. The frequency of these steps will be kept small enough so that they are essentially masked by the long-term dispersion of the clock ensemble.

At this writing the accuracy rate of AT(NBS) using eq (9.36) is 5.2 parts in 10^{13}, $\sigma_b(l)$ is estimated at about 5 parts in 10^{13}, and the best estimate of the rate of AT(NBS) is too low by 4.6 parts in 10^{13} with respect to NBS-5.

9.5. THE UTC(NBS) COORDINATED SCALE

The atomic time scale AT(NBS) is an independent and proper[4] time scale. The readings of any two such scales will have a comparative dispersion for two fundamental reasons: first, from deterministic differences; and second, from the random noise inherent in any time scale system. The first reason for dispersion may be caused by differences in accounting for the biases affecting the primary frequency standards of each scale; these differences should fall within the ascertained accuracy limits. Or it may be caused by the difference in gravitational potential at which the two scales are running ($-gH/c^2$; H is the differential height). The second reason for dispersion may be caused by fundamental noise processes in the clocks and/or by noise inserted by the measurement system or while the data are being processed in a particular time scale algorithm; i.e., different algorithms will have different amounts of dispersion.

Because of the above mentioned dispersion, if two different time scales are to be kept synchronized, it becomes necessary to insert a rate and a time correction in one or the other or in both time scales. Starting on 1 October 1968 a mutual coordination agreement was made with the United States Naval Observatory (USNO) to keep the UTC(USNO) and UTC(NBS) scales synchronized to within about 5 μs by making equal and opposite coordinated rate corrections [25].

More recently the philosophy that NBS has adopted is to generate a nonindependent coordinated time scale UTC(NBS) which is kept synchronous with the internationally adopted time scale maintained at the Bureau International de l'Heure (BIH) – denoted UTC [6]. This is accomplished as indicated in figure 9.12 with slight rate and leap second additions to AT(NBS). The constant C was chosen so that at 00 h 00 min 00 s UTC on 1 January 1972 the UTC(NBS) scale was synchronous to within about 3 μs of UTC. Because of similar direc-

[4] Proper is here used in the relativistic sense.

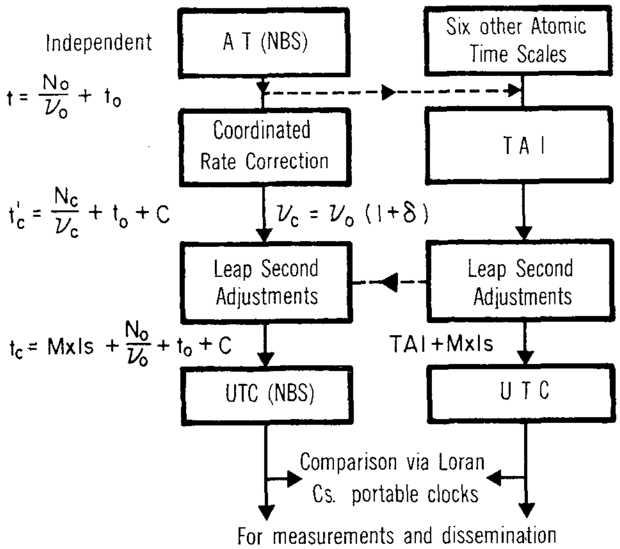

FIGURE 9.12. Block diagram showing the derivation of UTC(NBS) from AT(NBS) and its coordinated relationship to UTC(TAI).

tions taken by the USNO to keep their UTC(USNO) scale synchronized to UTC after 1 January 1972 the above agreement to keep nominal synchronization will continue to be maintained to within a few microseconds.

The insertion of leap second adjustments is determined by the BIH following CCIR regulations [25]. The insertion usually occurs on 30 June and/or 31 December for the purpose of keeping the time difference | UTC−UT1 | nominally less than 0.7s. M was set to minus 10 on 1 January 1972, minus 11 on 30 June 1972, minus 12 on 31 December 1972, and minus 13 on 31 December 1973.

The equation relating AT(NBS) and UTC(NBS) as of 1 May 1973 is given by:

$$\begin{aligned} \text{AT(NBS)} - \text{UTC(NBS)} &= 12.045153000\text{s} \\ &\quad - (129.6\ \text{ns/day}) \cdot (\text{MJD} - 41803\ \text{days}), \end{aligned} \qquad (9.37)$$

where MJD is the Modified Julian Day number. Equation (9.37) does not account for second order terms; i.e., atomic days are not used. (Recent investigations indicate that we need to define an Atomic Julian Day count or its modified equivalent for consistency so that eq (9.37) can be made exact.) It is anticipated that the δ_{NBS} coordinate rate corrections will be made on 1 January of each year in order to keep the time difference | UTC−UTC(NBS) | as small as practicable while also maintaining close coordination with UTC(USNO).

9.6. UTC(NBS) ACCESSIBILITY

The determination of a date on the AT(NBS) scale or on the coordinated UTC(NBS) scale requires some method of accessibility. Figure 9.13 gives a fractional frequency stability, $\sigma_y(\tau)$ versus τ plot for the main methods of communicating the time and/or frequency of these two scales. The stability data for figure 9.13 were taken from references [9, 26–29] or were measured and computed by the authors. Typically, the UTC(NBS) scale is the one disseminated and eq (9.37) or its updated version, which is published in reference [30], is required to compute AT(NBS). For comparison purposes the estimated stability of AT(NBS), which will be essentially the same as the stability for UTC(NBS), and the calculated and measured stability of NBS–X4 and of NBS–5 respectively are also plotted in figure 9.13. It is anticipated that the stability of NBS–5 will improve from further advancements in both its electronic servo system and its beam optics.

The date on the UTC(NBS) scale is communicated from NBS/Boulder, CO to the NBS radio stations (WWV, WWVB, and WWVL) near Ft. Collins, CO via the TV line-10 time transfer system [7, 9, 11]. Notice the excess noise (day to day) on the accumulated time error plot for clock 8 in figure 9.11. This is one of the three cesium clocks at WWV which is used in the AT(NBS) clock ensemble. This excess noise is in the TV line-10 time transfer system and amounts to $\langle \sigma_x^2(2, T=1\,d, \tau \simeq 1/f_h, f_h) \rangle^{1/2} \simeq 30$ ns. Time and/or frequency are communicated to WWVH at Kekaha, Kauai, Hawaii via WWVB transmissions and portable clocks.

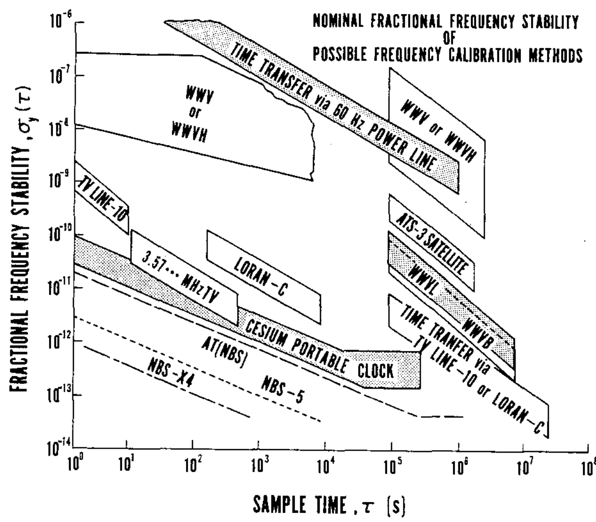

FIGURE 9.13. Fractional frequency stability as a function of sample time for several UTC(NBS) dissemination techniques.

Note that the signals from Loran–C, 60-Hz power line, as well as TV line-10 need to be used in the time transfer mode; i.e., the date of arrival of the signal on the UTC(NBS) scale must be differenced with the date of arrival of the signal ascertained with the user's clock in order to determine if a change has occurred in the user's clock time since the last measurement. In the case of the latter two the path delay has to be calibrated for time transfer (not frequency transfer), e.g., with a portable clock.

The following relationship allows the conversion from the fractional frequency stability to the time stability for most of the methods shown in figure 9.13 (see ref. [18]):

$$\langle \sigma_x^2 (2, T, \tau_x, f_h) \rangle^{1/2} = \frac{\tau}{\sqrt{4-2^{\mu+2}}} \sigma_y(\tau), \qquad (9.38)$$

where $2 < \mu < 0$, $T = \tau$, $\tau_x \ll T$, and it will be remembered that $\sigma_y(\tau) \sim \tau^{\mu/2}$. Applying eq (9.38) to the long-term stability data for Loran–C and TV line-10 as an example yields ~ 2.5 ns $\tau^{1/3}$ $s^{-1/3}$ over the range 1 day $\lesssim \tau \lesssim 100$ days.

Table 9.2 has been compiled to show characteristics of methods of obtaining data on the UTC-(NBS) scale. There are additional factors that need to be considered from the users point of view such as reliability, greater skill required, and number of users that can be served (see chap. 10, sec. 10.6 or ref. [26]). In addition to the techniques listed in table 9.2, personnel at NBS/Boulder, CO also monitor other radio stations (NAA, GBR, NLK), and these may be used in the time transfer mode to communicate UTC(NBS) date as the readings are published monthly [30].

The users' cost effectiveness for the precision and accuracy achieved by the techniques of TV line-10 and the ATS–3 satellite, respectively, is very good. Insertion of an active time code on a line in the blanking interval of the TV transmissions has been experimentally tested [31, 32] and the method has the potential of being extremely cost effective for the excellent precisions ($< 1\ \mu s$) achievable; in addition, it shows accuracy of maintaining the date once the path has been calibrated. The ATS–3 date transfer method was experimental and terminated

TABLE 9.2. *UTC(NBS) Accessibility*

Method of access	Coverage	Times available	Nominal accuracy for date transfer	Additional equipment cost to user
WWV and WWVH	hemisphere	continuously	$\lesssim 1$ ms	~$200 to $2,000.
WWVB	North America	continuously	$\sim 50\ \mu s$	~$400 to $4,000.
WWVL	global	experimental	envelope $\sim 500\ \mu s$	~$4,000.
ATS–3 Satellite	hemisphere	experimental 1700 to 1715 UTC 2300 to 2345 UTC daily (terminated late 1973).	10 to 50 μs	~$150.
Telephone (303) 499–7111	North America	continuously	$\lesssim 0.03$ s	price of phone calls.
Portable Cs. Clock	global	per user's desire	$\sim 0.1\ \mu s$	~$19,000.
Portable Rb. Clock	global	per user's desire	$\sim 1\ \mu s$	~$12,000.
Loran–C (ground wave)	east of Rocky Mtns. (N. Am.), through Europe.	1500 UTC each work day.	$\lesssim 3\ \mu s$	~$5,000 to $10,000.
Line-10 Television	requires common reception of east USA network programs.	~ 1330 local time (Boulder) each work day.	need to calibrate path; precision ~ 1 ns $\tau^{1/2}$ $\tau > 33$ ms	~$100 to $500.
60-Hz Power Line	USA and some adjacent areas. (Verified between CO and CA.)	proposed	need to calibrate path; precision $\lesssim 1$ ms τ_0 17 ms $\lesssim \tau \lesssim 10^5$ s.	~$10 to $100.
Omega Navigation System	global	proposed	$< 10\ \mu s$	~$16,000.

late 1973; however, the tests have proven successful [12]. There are plans for an NBS experiment which would provide time from UTC-(NBS) continuously, using a Department of Commerce satellite.

The direct signal from the 60-Hz power line is not very stable; $\sigma_y(\tau) \approx 5 \times 10^{-5}$ for 17 ms $\lesssim \tau \lesssim 10^5$ s). However, the stability of the differential delay between two points has much greater stability. The 60-Hz power line stability data shown in figure 9.13 represent the differential path delay between Santa Clara, CA and Boulder, CO. A similar study was performed for the path between Ft. Collins, CO and Boulder, CO, and the stability was about 5 times better. This very inexpensive time transfer system has significant potential for users needing synchronization to about 1 ms [33].

An exotic time and frequency dissemination system of the future with a potential of nanosecond precision and accuracy of date transfer may employ a belt of geostationary satellites around the globe. Each satellite may have an rf communications transponder and a triggerable infrared pulsed laser radiating the earth. Using trilateration with a grid of synchronous ground station clocks the position of the satellite could be determined to a few centimeters, and thence communicated, along with the dates of occurrence of the laser's pulses, to the appropriate receiving equipment. The infrared seems desirable because of available bandwidth, accurately calculable path delay, less problems with cloud cover than with the visible, and some comparative cost considerations.

9.7. THE INTERNATIONAL ATOMIC TIME SCALE (TAI)

The TAI scale is constructed at the BIH using the input via Loran–C and TV time transfer techniques of several cesium standards located in many of the time and frequency laboratories throughout the world—including the standards used to generate AT(NBS) (see fig. 9.12). There is reason to believe that the TAI is stable to about 1×10^{-13} for σ_y ($\tau \sim 1$ year) and is probably the most stable reference time scale available [3], hence its use as a reference may provide reasonable comparisons among the primary frequency standards of these laboratories. The fractional frequency of TAI with respect to four of the evaluable primary frequency standards in the world are as follows: for PTB (Germany), CS1 [$+ 12 \pm 4$ (1σ)] $\times 10^{-13}$, March–July 1970 [34]; for NRC, Cs III [$+ 8 \pm 15$ (2σ)] $\times 10^{-13}$, July 2–November 9, 1970 [2, 35]; fo NBS, NBS–III [$+10 \pm 5$ (1σ)] $\times 10^{-13}$,[5] May 1969 [23], and NBS–5 [12 ± 5 (1σ)] $\times 10^{-13}$,[5] January–April 1973 [36].

[5] These values account for ~ 1.8 parts in 10^{13} gravitational red shift due to the elevation of Boulder, Colorado.

AT(NBS) was in rate agreement to within 1×10^{-13} of the NBS primary frequency standard NBS–III in May 1969. Since that date NBS–III has been disassembled; parts were used in the construction of NBS–5. The rate of TAI with respect to AT(NBS) as of January 1, 1973 was about $+11 \times 10^{-13}$ via Loran–C [37]. If TAI were perfect the preceding would imply a decrease in the rate of AT(NBS) of about $\sim 2 \times 10^{-13}$ in approximately 4 years.

Such a rate drift singularly considered would cause a time dispersion of about 25 μs over this same period. However, if in eq (9.7) $D_{\text{AT(NBS)}}$ is assumed zero and $R_{\text{AT(NBS)}}$ is -75 ns/day as was estimated in 1969 by Dr. Guinot (The Director of the BIH), then the calculated value using eq (9.7) differs only by about 5 μs from that measured over the same 4 years. (Note: AT(NBS) is not totally independent of TAI.)

In the past the primary consideration for the TAI has been uniformity, with accuracy of secondary importance. With the adoption of TAI scale by the 14th General Conference of Weights and Measures (CGPM) in 1971 [38], conforming to the definition of the SI second, there is added emphasis upon accuracy. A recent study by the BIH has shown that the TAI scale has a 50 percent probability of an annual frequency drift of about $|0.5 \times 10^{-13}|$ [5]. "Except through the participation of NRC the scale unit of TAI is not anchored to the Système International d'Unites (SI) second and may diverge indefinitely from it. Recalibration is necessary." [5] There is a general awareness of this problem, and recommendations have been made by Dr. Guinot and others for reasonable corrective procedures (see sec. 1.5 of chap. 1) [39, 5]. Principally, the BIH is seeking individual clock data in place of a constructed time scale of an individual laboratory; this obviates some errors from individual clock and/or time scale equipment failures, as well as differences in method of calculating the individual laboratory time, TA(i), used in the construction of TAI. Dr. Guinot also recommends frequent comparison of the TAI second with that generated by evaluable primary frequency standards. After initial calibration, it will probably be necessary to make a frequency adjustment to the TAI scale and thereafter apply an intentional frequency drift ($\sim \pm 1.0 \times 10^{-13}$/year) so that TAI agrees with evaluations of the SI second [5]. It is hoped that many of the concepts and algorithms developed in this chapter may be of help to the BIH staff in their important task of constructing a TAI scale in conformance with the SI unit of time.

9.8. CONCLUSIONS

The rate of the AT(NBS) (proper) scale serves as a memory of the rate of the NBS primary frequency standard. The stability of the statistically weighted eight clock ensemble making up AT(NBS) determines the quality of the memory. The random fractional frequency fluctuations have been estimated and are reasonably modeled by:

$$\sigma^2_{y_e}(\tau) \simeq (5\times 10^{-12})^2\,\tau^{-1} + (3\times 10^{-14})^2, \qquad (9.39)$$

where $1\ \mathrm{s} \leqslant \tau \lesssim 10^7$ s. Some relative frequency drift has been observed between the members of the commercial cesium beam frequency standards composing the clock ensemble, and an estimate of the possible drift of the ensemble indicates that it should be $|D_e| \lesssim 2\times 10^{-13}$ per year. The accuracy (1σ) of the rate of AT(NBS) is currently estimated to be 5.2×10^{-13}.

The method employed for generating the AT(NBS) scale is based on a particular clock model. This model assumes that a linear frequency drift, a frequency offset, a time offset, and random noise can all perturb the time of a clock. A convenient recursive filter has been employed to process the data in a near optimum way, and to properly filter the perturbations introduced by all but the frequency drift; this drift can be measured over a sustained period with a primary frequency standard and can then be properly taken into account.

The UTC(NBS) scale differs in rate from the independent AT(NBS) scale by coordinated rate corrections; such corrections keep UTC(NBS) in nominal synchronization (to within a few μs) with UTC (the international Coordinated Universal Time scale maintained at the BIH). The clocks composing the AT(NBS) scale provide some of the input data for the time scale algorithm which generates TAI (UTC is kept within about 0.7 s of the UT1 scale by making leap second adjustments of UTC with respect to TAI).

Time and frequency are accessible from the UTC(NBS) scale via sundry dissemination and time communication methods; e.g., HF and LF radio (WWV, WWVH, WWVB, WWVL), telephone, portable clocks, Loran–C, line-10 TV and network TV color subcarrier frequency. These differ in accuracy of date transfer from ~ 50 ms to a few nanoseconds, and in the precision of a frequency calibration from parts in 10^7 to a few parts in 10^{14}. Nominally, the quality of precision is commensurate with equipment cost; i.e., higher precision requires increased equipment cost to the user. (TV and some satellite methods give the minimum cost per precision and accuracy achievable.)

The fractional frequencies of the TAI have been determined with respect to four of the world's evaluable primary frequency standards. TAI is probably the most stable time scale in the world with a stability of about 1×10^{-13} for $\sigma_y(\tau \sim 1$ year) [3]; it has been reported that this scale shows frequency drifting of some $|0.5\times 10^{-13}|$ per year [5]. The BIH is proposing several solutions for improving this free running scale and making it more nearly in agreement with the SI unit of time. Some of the techniques discussed in the chapter could aid conceivably in the evaluation and construction of the TAI. Possible means for comparing time scales include portable clocks [9], aircraft flyover [40], satellite/television techniques [41], and Loran–C [9].

AT(NBS) was in rate agreement to within 1×10^{-13} of NBS–III in May 1969. The rate of TAI with respect to AT(NBS) as of January 1973 was about 11×10^{-13} via Loran–C. If TAI were perfect this implies a decrease in the rate of AT(NBS) of $\sim 2\times 10^{-13}$ over approximately 4 years; this is well within the uncertainties internally assigned to the AT(NBS) system. The accuracy goal for NBS–5 is 1×10^{-13}. Attainment of this goal obviously will allow a significant improvement in the accuracy of the AT(NBS) rate. A preliminary calibration with NBS–5 of AT(NBS) was made during January–April 1973; the rate measured for AT(NBS) was within 1×10^{-13} of NBS–5's rate. As of March 1973, TAI appeared to be too high in rate by 12×10^{-13} $\pm 5\times 10^{-13}$ (the current accuracy estimate for AT(NBS) and NBS–5) with respect to NBS–5 and via a filtered estimate using Loran–C data as a time transfer mechanism.

9.9. REFERENCES

[1] Winkler, G. M. R., Hall, R. G., and Percival, D. B., "The U.S. Naval Observatory clock time reference and the performance of a sample of atomic clocks," *Metrologia*, **6**, No. 4, pp. 126–134 (October 1970).

[2] Mungall, A. G., "Atomic time scales," *Metrologia*, **7**, No. 4, pp. 146–153 (October 1971).

[3] Allan, D. W., "Statistical modeling and filtering for optimum atomic time scale generation," *Proc. Frequency Standards and Metrology Seminar* (Quebec, Canada, August 30, 31–September 1, 1971) H. Hellwig (Ed.), pp. 388–410 (Dept. Elec. Eng., Laval Univ., Quebec, Canada, June 1972).

[4] Yoshimura, K., "The generation of an accurate and uniform time scale with calibrations and prediction," *Nat. Bur. Stand. (U.S.) Tech. Note 626*, 64 pages (USGPO, November 1972).

[5] Guinot, B. and Granveaud, M., "Atomic time scales," *IEEE Trans. on Instrum. and Meas.*, **IM-21**, No. 4, pp. 396–400 (November 1972).

[6] CCIR, "Detailed instructions by Study Group 7 for the implementation of Recommendation 460 concerning the improved coordinated universal time (UTC) system, valid from 1 January 1972," *in XIIth Plenary Assembly CCIR* (New Delhi, India, 1970), **III**, pp. 258a–d (ITU: Geneva, Swtizerland, 1970) (see chap. 1-ann. 1.B.2).

[7] Milton, J. B., "Standard time and frequency: its generation, control, and dissemination from the National Bureau of Standards Time and Frequency Division," *Nat. Bur. Stand. (U.S.) Tech. Note 379*, 27 pages, (USGPO, August 1969) (see future updates).

[8] Viezbicke, P. P. (Ed.), "NBS frequency and time broadcast services, radio stations WWV, WWVH, WWVB, WWVL," *Nat. Bur. Stand. (U.S.) Misc. Pub. 236*, 19 pages (USGPO, May 1973). Note: A new edition is published annually.

[9] Allan, D. W., Blair, B. E., Davis, D. D. and Machlan, H. E., "Precision and accuracy of remote synchronization via network television broadcasts, Loran–C and portable clocks," *Metrologia*, **8**, No. 2, pp. 64–72 (April 1972).

[10] Davis, D. D., "Frequency standard hides in every color set," *Electronics*, **44**, No. 10, pp. 96–98 (May 10, 1971).

[11] Davis, D. D., Blair, B. E., and Barnaba, J. F., "Long term continental U.S. timing system via television network, *IEEE Spectrum*, **8**, No. 8, pp. 41–52 (August 1971).

[12] Hanson, D. W., Hamilton, W. F., and Gatterer, L. E., "The NBS frequency and time satellite experiment using ATS-3, *Proc. 3rd Ann. Precise Time and Time Interval (PTTI) Strategic Planning Meeting* (Washington, DC 20390, November 16–18, 1971), pp. 155–165 (USNO, Washington, DC 20390, January 1972), (NTIS, AD 758739)
[13] CGPM, *Comptes Rendus des Séances de la Treizième Conférence Générale des Poids et Mesures* (Proceedings of the Sessions of the 13th General Conference of Weights and Measures) (Paris, France, October 1968), p. 103 (Gauthier-Villars, Paris, France 1968) (in French—see chap. 1-ann. 1.A.1.).
[14] Hellwig, H., "Frequency standards and clocks: a tutorial introduction," *Nat. Bur. Stand. (U.S.) Tech. Note 616*, 69 pages (USGPO, April 1972).
[15] Allan, D. W., "Statistics of atomic frequency standards," *Proc. IEEE*, **54,** No. 2, pp. 221–230 (February 1966).
[16] Barnes, J. A., Chi, A. R., Cutler, L. S., Healey, D. J., Leeson, D. B., McGunigal, T. E., Mullen, J. A., Jr., Smith, W. L., Snydor, R., Vessot, R. F. C., and Winkler, G. M. R., "Characterization of frequency stability," *IEEE Trans. on Instrum. and Meas.*, **IM-20,** No. 2, pp. 105–120 (May 1971) (see chap. 8).
[17] Barnes, J. A., "Tables of bias functions, B_1 and B_2, for variances based on finite samples of processes with power law spectral densities," *Nat. Bur. Stand. (U.S.) Tech. Note 375*, 37 pages (USGPO, January 1969) (see chap. 8).
[18] Barnes, J. A., "Atomic timekeeping and the statistics of precision signal generators," *Proc. IEEE*, **54,** No. 2, pp. 207–220 (February 1966).
[19] Vessot, R. F. C., Mueller, L. F., and Vanier, J., "The specification of oscillator characteristics from measurements made in the frequency domain," *Proc. IEEE*, **54,** No. 2, pp. 199–207 (February 1966).
[20] Barnes, J. A. and Allan, D. W., "A statistical model of flicker noise," *Proc. IEEE*, **54,** No. 2, pp. 176–178 (February 1966).
[21] Becker, G., "Mathematische Filterung von Messwerten: Anwendurgen im Bereich von Zeit und Frequenz" (Mathematical filtering of measuring values; applications in the area of time and frequency), *PTB-MITT*, **79,** No. 4, pp. 247–252 (August 1969) (in German).
[22] Barnes, J. A., and Jarvis, S. J., Jr., "Efficient numerical and analog modeling of flicker noise processes," *Nat. Bur. Stand. (U.S.) Tech. Note 604*, 22 pages (USGPO, June 1971).
[23] Glaze, D. J., "Improvements in atomic cesium beam frequency standards," IEEE Trans. on Instrum. and Meas., **IM-19,** No. 3, pp. 156–160 (August 1970) (see chap. 5A).
[24] Risley, A. S., Allan, D. W., Gray, J. E., Shoaf, J. H., Machlan, H. E., Glaze, D. J., Peters, H. E., Johnson, E. H., Vessot, R. F. C., and Levine, M. W., "Long term frequency stability of a NASA prototype hydrogen maser" (Summary), *CPEM Digest* (1970 Conf. on Precision Electromagnetic Meas., Boulder, CO 80302, June 2–5, 1970), pp. 74–75 (IEEE, New York, NY 10017, 1970).
[25] Allan, D. W. and Gray, J. E., "Comments on the October 1970 Metrologia Paper 'The U.S. Naval Observatory clock time reference and the performance of a sample of atomic clocks'," *Metrologia*, **7,** No. 2, pp. 79–82 (April 1971).
[26] Jespersen, J. E., Blair, B. E., and Gatterer, L. E., "Characterization and concepts of time-frequency dissemination," *Proc. IEEE*, **60,** No. 5, pp. 502–521 (May 1972) (see chap. 10).
[27] Watt, A. D., Plush, R. W., Brown, W. W., and Morgan, A. H., "Worldwide VLF standard frequency and time signal broadcasting," *J. Research Nat. Bur. Stand.* (U.S.) (Radio Propagation), **65D,** No. 6, pp. 617–627 (November/December 1961).
[28] Morgan, A. H., "Distribution of standard frequency and time," *Proc. IEEE*, **55,** No. 6, pp. 827–836 (June 1967).
[29] Winkler, G. M. R., "Path delay, its variations, and some implications for the field use of precise frequency standards," *Proc. IEEE*, **60,** No. 5, pp. 522–529 (May 1972).
[30] Viezbicke, P. P. (Ed.), "NBS time and frequency services bulletin" (Published monthly by the Frequency-Time Broadcast Services Section, NBS, Boulder, CO 80302).
[31] Davis, D. D., Jespersen, J. L., and Kamas, G., "The use of television signals for time and frequency dissemination," *Proc. IEEE* (Lett.), **58,** No. 6, pp. 931–933 (June 1970).
[32] Howe, D. A., "Nationwide precise time and frequency distribution utilizing an active code within network television broadcasts," *IEEE Trans. on Instrum. and Meas.*, **IM-21,** No. 3, pp. 263–276 (August 1972).
[33] Allan, D. W., Machlan, H. E., and Marshall, J., "Time transfer using nearly simultaneous reception times from a common transmitter," *Proc. 26th Ann. Symp. on Frequency Control* (U.S. Army Electronics Command, Ft. Monmouth, NJ 07703, June 6–8, 1972), pp. 309–316 (Electron. Ind. Assn., Washington, DC 20006, 1972).
[34] Becker, G., Fischer, B., Kramer, G., and Muller, E. K., "Wissenschaftliche Abhandlungen der Physikalisch-Technischen Bundesanstalt" (Scientific Trans. of the PTB), **22,** pt. 1, pp. 41–42 (1970) (in German).
[35] Mungall, A. G., "The second order Doppler shift in cesium beam atomic frequency standards, *Metrologia*, **7,** No. 2, pp. 49–56 (April 1971).
[36] Allan, D. W., Glaze, D. J., Machlan, H. E., Wainwright, A. E., Hellwig, H., Barnes, J. A., and Gray, J. E., "Performance, modeling, and simulation of some cesium beam clocks," *Proc. 27th Ann. Symp. on Frequency Control* (U.S. Army Electronics Command, Ft. Monmouth, NJ 07703, June 12–14, 1973), pp.334–346 (Electron. Ind. Assn., Washington, DC 20006, 1973).
[37] Computed from the Bureau International de l'Heure, Cir. D. Bull. (BIH, Paris, France, 1972).
[38] CGPM, *Comptes Rendus des Séances de la Quatorzième Conférence Générale des Poids et Mesures* (Proceedings of the Sessions of the 14th General Conference of Weights and Measures) (Paris, France, October 4–8, 1971), pp. 49–52; 77–78 (BIPM, Sevres, France, 1972) (in French—see chap. 1-ann. 1.A.3).
[39] Smith, H. M., "International time and frequency coordination," *Proc. IEEE*, **60,** No. 5, pp. 479–487 (May 1971).
[40] Besson, J., "Comparison of national time standards by simple overflight," *IEEE Trans. on Instrum. and Meas.*, **IM-19,** No. 4, pp. 227–232 (November 1970).
[41] Palii, G. N. and Fedorov, Y. F., "Method of relating time scales with the aid of a satellite link and terrestrial television networks" (translated from Izmeritel'naya Tekhnika, No. 3, pp. 23–26, March 1972), *Meas. Tech. (USSR)*, **15,** No. 3, pp. 380–384 (Consultants Bureau, New York, NY 10017, March 1972).

ANNEX 9.A.

OPTIMUM FILTERS FOR VARIOUS NOISE PROCESSES (Eq (9.19))

This Annex considers in detail optimum filters for noise processes in eq (9.19) as follows:

$$\mu = \begin{cases} -2, \text{for } \alpha \geqslant 1 \text{ and } \tau f_h \geqslant 1; \\ -\alpha - 1, \text{for } -3 < \alpha < 1. \end{cases} \tag{9.19}$$

(Optimum, as used here, gives a minimum squared error of time prediction $\langle \epsilon^2_{\text{opt}}(\tau) \rangle$.)

The cases for $\alpha = 2$, 0, -2 are straightforward. For $\alpha = 2$, white noise PM, the time fluctuations are already a white noise process; hence, the simple mean of the residual time fluctuations is the optimum predictor, and the standard deviation of the mean is an estimate of $\langle\epsilon^2(\tau)\rangle$. In timekeeping practice this predictor has not been very useful since white noise PM usually predominates in high performance clocks for times only much shorter than 1 second.

For $\alpha=0$—white noise FM or random walk of phase (time)—the random walk time fluctuations may be reduced easily to a white noise process by taking the first finite difference for discrete data $(\Delta x(j))$ or the time derivative $\left(\frac{dx(t)}{dt}\right)$ for the continuous case. The simple mean of either of these over the data set is the optimum predictor. This is equivalent to predicting the time from the residual rate given in eq (9.15), i.e., using the residual times at the beginning and end of the data set. The mean-square time dispersion is given by eq (9.18) where τ_c is the time interval over the data set. Obviously τ_c should be kept much larger than τ, the prediction interval, for optimum data usage where white noise FM is the predominate random perturbation. This noise process predominates in many frequency standards and in most cesium beam frequency standards for values of τ at least in the range $1\ \mathrm{s} \lesssim \tau \lesssim 10^5$ s—see figure 9.1. Hence, this noise process is fundamental in atomic timekeeping.

For $\alpha = -2$—random walk FM reduces to a white noise process by taking the second finite difference for equally spaced discrete data $(\Delta^2x(j))$ or the second time derivative $\left(\frac{d^2x(t)}{dt^2}\right)$ for the continuous case. For the discrete case the optimum time prediction at $t+\tau$ for the random fluctuations would then be given by:

$$\hat{x}(t+\tau)=\overline{\Delta^2x}+2x(t)-x(t-\tau), \qquad (9.A.1)$$

where $\overline{\Delta^2x}$ is the average over the data set. The mean-square time error is given by:

$$\langle\epsilon^2(\tau)\rangle=2\tau^2\sigma_y^2(\tau), \qquad (9.A.2)$$

which is also equal to $\langle(\Delta^2x)^2\rangle$.

For $\alpha=1$ and $\alpha=-1$—flicker noise PM and its integral flicker noise FM—the optimum prediction problem is more sophisticated. Recently some work has been done showing how flicker noise can be efficiently generated from white noise [21]; Barnes and Jarvis have pointed out that such a filter could be used in reverse to transform flicker noise into white noise. Once the appropriate averaging and inverse transformations were taken, one would have, in principle, an optimum flicker filter over an arbitrary number of decades of τ. We are studying this technique at the present time; however, in the past we have investigated some very simple recursion relationships which yield near optimum time prediction (see sec. 9.3.2, eq (9.20)).

ANNEX 9.B.

TIME DISPERSION WITH RECURSIVE FILTER APPLIED TO FLICKER NOISE FM

The second term on the right of eq (9.25) gives the mean-squared time error after an interval τ when a recursive filter, as in eq (9.20), is applied to a flicker noise FM process of intensity h_{-1}. Such a filter is approximately equal to an exponential filter having a time constant $m\tau$. Since from table 9.1 we have the following

$$\sigma_y^2(\tau)=2h_{-1}\ln(2), \qquad (9.B.1)$$

the mean-squared time error due to h_{-1} in eq (9.25) may be rewritten as:

$$\langle\epsilon^2(\tau)\rangle=\sigma_y^2(\tau)F(m)\cdot\tau^2, \qquad (9.B.2)$$

where

$$F(m)=\frac{1}{2\ln(2)m^2(2m+1)}\sum_{k=1}^{\infty}\left(\frac{m}{m+1}\right)^k k^2\ln k. \qquad (9.B.3)$$

Table 9.B.1 is a list of some pertinent values for $F(m)$. The time dispersion due to flicker noise FM over an interval τ may be easily calculated (the square root of eq (9.B.2)) by use of table 9.B.1 once we know the stability $\sigma_y(\tau)$, and the value of m.

TABLE 9.B.1. *Correspondence of Values* m *and* F(m) *in eq (9.B.3)*

$$F(m)=\frac{1}{2\ln(2)m^2(2m+1)}\sum_{k=1}^{\infty}\left(\frac{1}{m+1}\right)^k k^2\ln k \qquad (9.B.3)$$

m	$F(m)$	m	$F(m)$
0	2.00	10	2.60
0.1	1.93	20	2.99
0.2	1.89	30	3.23
0.3	1.86	40	3.42
0.4	1.85	50	3.56
0.5	1.85	60	3.69
0.6	1.84	70	3.79
0.7	1.85	80	3.88
0.8	1.85	90	3.96
0.9	1.86	100	4.03
1.0	1.87	200	4.51
2	1.97	300	4.80
3	2.08	400	5.00
4	2.18	500	5.16
5	2.27	600	5.29
6	2.35	700	5.40
7	2.42	800	5.49
8	2.48	900	5.59
9	2.54	1000	5.65

ANNEX 9.C.

MINI-COMPUTER PROGRAM FOR FIRST ORDER TIME SCALE ALGORITHM

This program is written in a Fortran dialect compatible with a mini-computer which has a 4k twelve-bit word memory. The computation has been simplified with Fortran variables redefined, in an effort to work within the capabilities of the mini-computer. The program of computation is explained by the Fortran listing with its comment statements. In these statements the computed time scale is often called the "paper clock."

Input required by the computer program:

DAY2 — If initial values are to be supplied, DAY2 is 9, otherwise it is the date on which the clock readings were taken, i.e., the modified Julian day number.

FN(I) — is the number of measurement intervals to be used as a time constant when computing a new rate for clock I.

R(I) — is the rate of clock I with respect to that of the paper clock in nanoseconds per day.

D(I) — is the time of clock I with respect to the paper clock in nanoseconds; D(I) = clock I − paper clock.

SIG(I) — is $\langle \epsilon_i^2(\tau=1\mathrm{d}) \rangle^{1/2}$ for clock I in nanoseconds.

SUME(I) — is the total time error (nanoseconds) accumulated by clock I, with respect to the paper clock, since sum error was last set equal to zero.

DAY1 — is the date on which the last set of readings were taken.

T — is the hour at which the readings were taken. In this program, *T* is not used in the computation, the readings being taken at the same time each day.

D31 — is the time interval, in nanoseconds, between the 1 pps of clock 3 and that of clock 1, etc. for all the other clock differences entered into the program.

D3S — is the time interval, in nanoseconds, between the 1 pps of clock 3 and that of clock S. Clock S is the on-line clock which approximates UTC(NBS)—often called the working standard.

Interpretation of the computer printout:

(1) Modified Julian day: The date.
(2) Clock number: The clocks of the ensemble are numbered 1 through 8.
(3) Error: The error in nanoseconds, accumulated by clock I during the measurement interval, with respect to the paper clock.
(4) Frequency: The frequency of clock I with respect to that of the paper clock, in parts in 10^{14}, averaged over the measurement interval.
(5) Sigma of the set: Corresponds to $\langle \epsilon_{\mathrm{e}}^2(\tau=1\mathrm{d}) \rangle^{1/2}$ in eq (9.25).
(6) Time of the working standard: The time of clock S with respect to the paper clock in nanoseconds.

The remaining output is data that is to be put back into the computer for computation on the next day.

In case of clock reset, the following are unchanged: sum error, rate, and sigma. The "Error" printout has the same significance as usual, and the rate is unadjusted.

Fortran listing follows:

```
C;      TIME SCALE PROGRAM (15 MARCH 71). A MINUS (-) INDICATES THAT
C;      THE CLOCK IS EITHER LOW IN FREQUENCY OR LATE IN TIME.
1;      FORMAT(I,E,E,E,E,/)
2;      FORMAT("CLOCK ",I,"RESET",/)
3;      FORMAT(//,"SMPL AVG  RATE              TIME            SIGMA
4;      FORMAT(E)
5;      FORMAT(I,E,E,/)
6;      FORMAT(/,"MODIFIED JULIAN DAY ",E,/)
7;      FORMAT(/////)
8;      FORMAT("TIME OF WORKING STANDARD IS ",E,/)
9;      FORMAT(/,"SIGMA OF SET  ",E,/)
        DIMENSION R(8), D(8), SIG(8), ER(8), DP(8), IRJT(8), SUME(8),FN(8
11;     FORMAT(/,"CLOCK     ERROR      FREQ X 10 TO 14",/)
        N1=1
        N8=8
10;     ACCEPT 4, DAY2
        IF (DAY2-9.) 30,30,40
30;     DO 39 I=N1,N8
        ACCEPT 4,FN(I), R(I), D(I),SIG(I), SUME(I)
39;     CONTINUE
        ACCEPT 4, DAY1
        GO TO 10
40;     ACCEPT 4, T,D31,D32,D34,D35,D84,D86,D37,D3S
        T=DAY2-DAY1
        DO 49 I=N1,N8
        DP(I)=R(I)*T + D(I)
        IRJT(I)=0
49;     CONTINUE
        N=0
50;     ER(1)=DP(1)-D3S+D31
        ER(2)=DP(2)+D32-D3S
        ER(3)=DP(3)-D3S
        A=D34-D3S
        ER(4)=DP(4)+A
        ER(5)=DP(5)+D35-D3S
        ER(6)=DP(6)+A-D84+D86
        ER(7)=DP(7)-D3S+D37
        ER(8)=DP(8)+A-D84
        ANRM=0.
        DS=0.
        DO 69 I=N1,N8
        IF (IRJT(I)-N) 68,68,69
```

```
68;     DS=DS+ER(I)/SIG(I)**2
        ANRM=ANRM+1./SIG(I)**2
69;     CONTINUE
        DS=DS/ANRM
        DAY1=0.
        DO 79 I=N1,N8
        ER(I)=DS-ER(I)
        DAY1=DAY1+ER(I)*ER(I)
79;     CONTINUE
        M=0
        DO 89 I=N1,N8
        A=ER(I)*ER(I)-.2*DAY1-4.E4
        IF (A) 86,87,87
86;     A=(ER(I)/T)*(ER(I)/T) - 9.*SIG(I)**2
        IF (A) 88, 85, 85
87;     IRJT(I)=9
        GO TO 84
88;     IRJT(I)=0
        GO TO 89
85;     IRJT(I)=N8
84;     M=M+1
89;     CONTINUE
        IF (M-N) 60,90,60
60;     N=M
        GO TO 50
90;     TYPE 6, DAY2
        TYPE 11
        ANRM=SQTF(1./ANRM)
        DO 98 I=N1,N8
        DAY1=DP(I) + ER(I)
        IF (IRJT(I)) 10,91,96
91;     SUME(I)=SUME(I)+ER(I)
        A=(DAY1-D(I))/(T*.864)
        R(I)=(R(I)*FN(I)+.864*A)/(FN(I)+1.)
        ERNM=SQTF(ER(I)*ER(I))/T+.8*ANRM**2/SIG(I)
        C=(31.*SIG(I)**2+ERNM**2)/32.
        SIG(I)=SQTF(C)
        GO TO 97
96;     TYPE 2, I
97;     D(I)=DAY1
        TYPE 5, I, ER(I), A
98;     CONTINUE
        TYPE 9, ANRM
        TYPE 8, DS
        TYPE 3
        DO 119 I=N1,N8
        TYPE 1, FN(I), R(I), D(I), SIG(I), SUME(I)
119;    CONTINUE
        TYPE 7
        DAY1=DAY2
        GO TO 10
        END
```

ANNEX 9.D.

SELECTED BIBLIOGRAPHY ON TIME SCALES AND THEIR FORMATION

[B1] Allan, D. W., "Statistics of atomic frequency standards," *Proc. IEEE*, **54,** No. 2, pp. 221–230 (February 1966).

[B2] Allan, D. W., "Statistical modeling and filtering for optimum atomic time scale generation," *Proc. Frequency Standards and Metrology Seminar* (Laval University, Quebec, Canada, August 30, 31, September 1, 1971), H. Hellwig (Ed.), pp. 388–410 (Dept. Elec. Eng., Laval Univ., Quebec, Canada, June 1972).

[B3] Allan, D. W., Fey, R. L., Machlan, H. E., and Barnes, J. A., "An ultra-precise time synchronization system designed by computer simulation," *Frequency*, **6,** No. 1, pp. 11–14 (January 1968).

[B4] Allan, D. W., Gray, J. E., and Machlan, H. E., "The NBS atomic time scale system: AT(NBS), SAT(NBS), and UTC(NBS)," (summary) *Proc. 24th Ann. Symp. on Frequency Control* (U.S. Army Electronics Command, Ft. Monmouth, NJ 07703, April 27–29, 1970), p. 361 (Electron. Ind. Assn., Washington, DC 20006, 1970).

[B5] Allan, D. W., and Gray, J. E., "Comments on the October 1970 Metrologia Paper The U.S. Naval Observatory clock time reference and the performance of a sample of atomic clocks," *Metrologia*, **7,** No. 2, pp. 79–82 (April 1971).

[B6] Allan, D. W., Gray, J. E., and Machlan, H. W., "The National Bureau of Standards atomic time scales: generation, dissemination, stability, and accuracy," (*Proc. of the 1972 Conf. on Precision Electromagnetic Meas. (CPEM)*, Boulder, CO 80302, June 26–29, 1972); *IEEE Trans. on Instrum. and Meas.*, **IM-21,** No. 4, pp. 388–391 (November 1972).

[B7] Barnes, J. A., "The development of an international atomic time scale," *Proc. IEEE*, **55,** No. 5, pp. 822–826 (June 1967).

[B8] Barnes, J. A., and Allan, D. W., "An approach to the prediction of coordinated universal time," *Frequency*, **5,** No. 6, pp. 15–20 (November–December 1967).

[B9] Barnes, J. A., Andrews, D. H., and Allan, D. W., "The NBS–A time scale—its generation and dissemination," *IEEE Trans. on Instrum. and Meas.*, **IM-14,** No. 4, pp. 228–232 (December 1965).

[B10] Barnes, J. A., Chi, A. R., Cutler, L. S., Healey, D. J., Leeson, D. B., McGunigal, T. E., Mullen, J. A., Jr., Smith, W. L., Sydnor, R. L., Vessot, R. F. C., and Winkler, G. M. R., "Characterization of frequency stability," *IEEE Trans. on Instrum. and Meas.*, **IM-20,** No. 2, pp. 105–120 (May 1971) (see chap. 8).

[B11] Barnes, J. A., and Fey, R. L., "Synchronization of two remote atomic time scales," *Proc. IEEE (Corresp.)*, **51,** No. 11, p. 1665 (November 1963).

[B12] Barnes, J. A., and Jarvis, S., Jr., "Efficient numerical and analog modeling of flicker noise processes," *Nat. Bur. Stand. (U.S.) Tech. Note 604*, 22 pages (USGPO, June 1971).

[B13] Becker, G., "Von der Astronomischen zur Atomphysikalischen Definition der Sekunde," (From the astronomical to the atomic definition of the second), *PTB-Mitt.*, **76,** No. 4, pp. 315–323 and No. 5, pp. 415–419 (August and Octover 1966) (in German).

[B14] Becker, G., "Mathematische Filterung von Messwerten; Anwendurgen im Bereich von Zeit und Frequenz," (Mathematical filtering of measuring values; applications in the area of time and frequency), *PTB-Mitt*, **79,** No. 4, pp. 247–252 (August 1969) (in German).

[B15] Becker, G., "Uber die Begriffe Phase, Phasenzeit und Phasenwinkel bei Zeitabhangigen Vorgungen," (About the terms phase, phase time, and phase angle in time dependent processes), *PTB-Mitt.*, **81,** No. 5, pp. 348–352 (October 1971) (in German).

[B16] Becker, G., "Was ist eine Zeitskala?," (What is a time scale?), *PTB-Mitt.*, **81,** No. 6, pp. 405–411 (December 1971) (in German).

[B17] Besson, J., "Comparison of national time standards by simple overflight," *IEEE Trans. on Instrum. and Meas.*, **IM-19,** No. 4, pp. 227–232 (November 1970).

[B18] BIPM (International Bureau of Weights and Measures), "Changement de la définition de la seconde," (Change of the definition of the second), *Comptes Rendus des Séances de la Triezième Conférence Générale des Poids et Mesures*, (Proc. Meet. of 13th General Conf. of Weights and Measures, October 10–16, 1967; October 15, 1968, Paris, France), pp. 58–60; Unit of time (second) resolution, p. 103 (Gauthier-Villars & Cie, Paris, France, 1969) (in French—see chap. 1-ann. 1.A.1).

[B19] Bodily, L. N., and Hyatt, R. C., "'Flying clock' comparisons extended to east Europe, Africa, and Australia," *Hewlett-Packard J.*, **19,** No. 4, pp. 12–20 (HP Co., Palo Alto, CA 94304, December 1967).

[B20] Bonanomi, J., Kartaschoff, P., Newman, J., Barnes, J. A., and Atkinson, W. R., "A comparison of the TA_1 and the NBS-A atomic time scales," *Proc. IEEE (Corresp.)*, **52,** No. 4, p. 439 (April 1964).

[B21] CCDS, "Recommendations du comité consultatif pour la définition de la seconde . . .," (Recommendations of the Consultative Committee for the definition of the second . . .), *Comité consultatif pour la Définition de la Seconde* (CIPM, 5^{e} Session, Paris, France, June 18–19, 1970), pp. S21–S23 (CIPM, Sèvres, France, 1971) (in French—see chap. 1-ann. 1.A.2).

[B22] CCIR, "Comparisons of time scales by various methods," CCIR Study Group 7, Doc. 7/8–E, Rep. 363–, *XIIth Plenary Assembly CCIR* (New Delhi, India, 1970), **III,** pp. 249a–b (ITU: Geneva, Switzerland, 1970).

[B23] CGPM, *Comptes Rendus des Séances de la Quatorzième Conférence Générale des Poids et Mesures* (Proc. of the Sessions of the 14th General Conference of Weights and Measures) (Paris, France, October 4–8, 1971), pp. 49–52; 77–78 (BIPM, Sèvres, France, 1972) (in French—see chap. 1-ann. 1.A.3).

[B24] CIPM (International Committee of Weights and Measures), *Comité Consultatif pour la Définition de la Seconde* (Consultative Committee for the Definition of the Second), 3^{e} Session–1963, 72 pages; 4^{e} Session–1967, 29 pages (Gauthier-Villars & Cie, Paris, France, 1964; 1968) (in French).

[B25] Essen, L., "Time scales," *Metrologia*, **4,** No. 4, pp. 161–165 (October 1968).

[B26] Essen, L., "A revolution in time-keeping," *Proc. of Conf. on Freq. Generation and Control for Radio Syst.* (C. E. Tate, Chairman, London, England, May 22–24, 1967), *Inst. Elec. Eng. (London) Conf. Publ. No. 31*, pp. 11–15 (Savoy Place, London, England, 1967).

[B27] Fey, R. L., Barnes, J. A., and Allan, D. W., "An analysis of low information rate time control unit," *Proc. 20th Ann. Symp. on Frequency Control* (U.S. Army Electronics Command, Ft. Monmouth, NJ 07703, April 19–21, 1966), pp. 620–635 (NTIS, AD 800 523, 1966).

[B28] Ganter, W. A., "Modeling of atomic clock performance and detection of abnormal clock behavior," *Nat. Bur. Stand. (U.S.) Tech. Note 636*, 30 pages (USGPO, March 1973).

[B29] Guinot, B., "Characteristics of a scale of time," *Alta. Frequenza*, **40,** No. 8, pp. 320E–322E (August 1971) (in English).

[B30] Guinot, B., Feissel, M., and Granveaud, M., *Bureau International de l'Heure Annual Report for 1970* (BIH, Paris, France, 1971).

[B31] Guinot, B., "Sixieme rapport due comité consultatif pour de la définition de la seconde au comité international des poids et mesures," *(Minutes of the 6th Session Consulta-*

tive Committee for the Definition of the Second) (CIPM, 6^e Session, Paris, France, July 6–7, 1972) (CIPM, Sèvres, France, 1972) (in French—see chap. 1-ann. 1.C.1.).

[B32] Guinot, B., and Granveaud, M., "Atomic time scales," *IEEE Trans. on Instrum. and Meas.*, **IM-21**, No. 4, pp. 396–400 (November 1972).

[B33] Hudson, G. E., "Some characteristics of commonly used time scales," *Proc. IEEE*, **55**, No. 6, pp. 815–921 (June 1967).

[B34] Kartaschoff, P., and Brandenberger, H., "A digital servo for frequency and time scale conversion," *Proc. 20th Ann. Symp. on Frequency Control* (U.S. Army Electronics Command, Ft. Monmouth, NJ 07703, April 19–21, 1966), pp. 577–586 (NTIS, AD 800 523, 1966).

[B35] Kovalevsky, J., "Astronomical time," Metrologia, **1**, No. 4, pp. 169–180 (October 1965).

[B36] Leschiutta, S., "The IEN time scale as an application of some ideas on timekeeping," *Proc. Frequency Standards and Metrology Seminar* (Laval University, Quebec, Canada, August 30, 31–September 1, 1971), H. Hellwig (Ed.), pp. 411–430 (Dept. Elec. Eng., Laval Univ., Quebec, Canada, June 1972).

[B37] Markowitz, W., "The atomic time scale," *IRE Trans. on Instrum.*, **I-11**, Nos. 3 and 4, pp. 239–242 (December 1962).

[B38] Milton, J. B., "Standard time and frequency: its generation, control and dissemination from the National Bureau of Standards Time and Frequency Division," *Nat. Bur. Stand. (U.S.) Tech. Note 379*, 27 pages (USGPO, August 1969) (see future updates).

[B39] Morris, D., Daams, H., Mungall, A. G., Bailey, R., Miller, L. G., and Thompson, M. M., "Atomic frequency standards and timekeeping at the National Research Council," *Proc. Frequency Standards and Metrology Seminar* (Laval University, Quebec, Canada, August 30, 31–September 1, 1971), H. Hellwig (Ed.) pp. 119–146 (Dept. Elec. Eng., Laval Univ., Quebec, Canada, June 1972).

[B40] Mungall, A. G., "Atomic time scales," *Metrologia*, **7**, No. 4, pp. 146–153 (October 1971).

[B41] Mungall, A. G., Daams, H., and Bailey, R., "Note on atomic timekeeping at the National Research Council," *Metrologia*, **5**, No. 3, pp. 73–76 (July 1969).

[B42] NBS, "Nation gets unified time system" *Nat. Bur. Stand. (U.S.) Tech. News Bull.*, **53**, No. 2, p. 34 (USGPO, February 1969).

[B43] Newman, J., Fey, R. L., and Atkinson, W. R., "A comparison of two independent atomic time scales," *Proc. IEEE(Corresp.)*, **51**, No. 3, pp. 498–499 (March 1963).

[B44] Putkovich, K., "Automated timekeeping," *IEEE Trans. on Instrum. and Meas.*, **IM-21**, No. 4, pp. 401–405 (November 1972).

[B45] Winkler, G. M. R., "Explanation and requirements for universal time," *Proc. PTTI (Precise Time and Time Interval Strategic Planning Meeting)*, (USNO, Washington, DC 20390, December 10–11, 1970), **1**, pp. 152–155 (NTIS, AD 881 014–L, 1970).

[B46] Winkler, G. M. R., Hall, R. G., and Percival, D. B., "The U.S. Naval Observatory clock time reference and the performance of a sample of atomic clocks," *Metrologia*, **6**, No. 4, pp. 126–134 (October 1970).

[B47] Yoshimura, K., "The generation of an accurate and uniform time scale with calibrations and predictions," *Nat. Bur. Stand. (U.S.) Tech. Note 626*, 64 pages (USGPO, November 1972).