Role of spurious reflections in ring-down spectroscopy

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Spurious coherent reflections from optical elements that re-enter an exit port of a two-mirror ring-down cavity can significantly change the effective reflectivity of the cavity mirrors, thus altering the cavity decay time. For a 25-cm-long Fabry–Perot cavity with a decay constant of 40 μs, we find that a specular reflection of only \(10^{-4}\) of the transmitted ring-down power that is mode matched back toward the cavity could change the decay time by as much as \(±0.4\) μs, depending on the phase of the returning reflection. The perturbation of the decay time is proportional to the electric field, so a decrease in the spurious reflected power of 100 times will result in a perturbation that is only 10 times smaller. We demonstrate the effect with a cw system by purposely introducing a spurious reflection.

\[ \tau^{-1} = \frac{\left[aL + \sum_{i=1}^{n} (1 - R_i)\right]}{nL/c}, \]  

where the intracavity loss \(a\) is in inverse centimeters, \(nL\) is the optical round-trip length in centimeters, and \(R_i\) is the power reflectivity of the \(i\)th mirror. Here we assume that the intracavity absorber fills the space between the mirrors. From Eq. (1) we may obtain the sensitivity of the power decay time constant to the intracavity absorption constant,

\[ \frac{d\tau}{da} = -\frac{c}{n} \tau^2, \]  

and also the intracavity absorption in terms of the decay time constant with and without the absorber present,

\[ a = \frac{n}{c} \left(\frac{1}{\tau} - \frac{1}{\tau_0}\right). \]  

We use our 25-cm-long \((L = 50\) cm\) two-mirror cavity with \(R_1 = R_2\) and a \(\tau \approx 40\) μs decay constant as an example to illustrate the problem of spurious reflections perturbing a decay measurement. Such a cavity has an absorption sensitivity, according to the inverse of Eq. (2), of \(\frac{da}{d\tau} \approx 2 \times 10^{-4}\) cm$^{-1}$/μs. The mirrors have a reflectivity \(R\) of approximately 0.999979. This reflectivity is obtained with a high-order thin-film design similar to that depicted in Fig. 1. The high reflectivity experienced by the cavity wave is due to the constructive interference of the reflections from each thin-film layer interface. However, the resultant reflectance of the cavity wave is also a function of what happens to the transmitted beam. If a fraction of this beam is returned to the cavity mode, the resultant field reflected from the mirror will be affected. It is the reflectance (reflected power/incident power), which is in fact the effective reflectivity, that is affected. For instance, whether the mirror is antireflection coated or not, the residual reflection from the back side of each high-finesse mirror will contribute a term that will manifest itself on the cavity time constant as a wavelength- (and temperature-) dependent cyclical baseline. However, other reflections from objects beyond the substrate’s back side are just as capable of changing the effective reflectivity and the subsequent cavity decay time constant. Such
reflections will cause the cavity decay time constant to depend on air currents, vibration, and temperature external to the cavity, since these items will modulate the phase of the returning light.

We analyzed this problem for our cavity by using thin-film design software to calculate the change in effective reflectivity caused by an interface some distance away from the thin-film stack. The index change at the interface was used to provide an adjustable reflection, the magnitude of which could be calculated from the Fresnel equations. Not surprisingly, the resultant mirror reflectivity plot versus wavelength clearly shows interference fringes with a period $c/2nl$, where $l$ is the distance to the interface. Maximum and minimum deviations from the nominal reflectivity (which could be caused by variation of either $n$ or $l$) were taken from these plots. The corresponding maximum deviations to be expected in the cavity decay time were then calculated with Eq. (1), with the nominal thin-film reflectivity for $R_1$ and the modified reflectivity for $R_2$. Plotted in Fig. 2 are the maximum deviations in the mirror's effective reflectivity from the nominal value caused by a given fraction of power reflected back toward the cavity.

The resultant effective reflectivity experienced by the cavity wave can be anything between these two extremes, depending on the relative phase of the spurious reflection. Furthermore, since it is a fraction of the transmitted cavity wave that is retroreflecting back that is perturbing the effective reflectivity of the high-finesse mirror, the perturbation is constant throughout the cavity decay, as long as the relative phase of the reflected light is constant. The resulting change in the time constant is also shown in Fig. 2. Although the reflectivity bounds are shown as two curves (since for large spurious retroreflections they are different), the positive- and negative-reflectivity bounds correspond to nearly symmetrical deviations from the nominal decay constant. Consequently, only one curve is shown, and it is labeled ± maximum deviation from the nominal decay.

A cyclical baseline that is due to the back surface of the high-finesse mirror substrate is the most obvious example of this effect. Our two mirrors are fused silica, approximately 4 mm thick, each of which should independently contribute a 25-GHz (0.83-cm$^{-1}$) period to the baseline of the cavity's decay constant. The contributions will tend to enhance or cancel each other out, depending on the phases of the returning back-surface reflections relative to the cavity wave. The cyclical baseline is temperature dependent (≈2.7 GHz/°C or 0.09 cm$^{-1}/°C$) because of the temperature dependence of the fused-silica refractive index and length. Figure 3 is an extreme example, in which the contributions to the cyclical baseline of two mirrors with uncoated substrates are partially in phase with each other. From Fig. 2, we expect the maximum change in the cavity decay time constant that is due to each mirror to be ±7.4 μs, or a potential deviation from the nominal cavity decay time by as much as ±14.8 μs. The measured change is ±5.5 μs, indicating that the effect of the two back-surface reflections on the cavity finesse may be partially out of phase.

We demonstrate this effect of spurious reflections affecting the cavity decay time by introducing a small external reflection, while varying the distance from the reflector to the cavity. A beam splitter ($R = 5.5\%$) was installed before the detector used to monitor the cavity transmission. The beam-splitter reflection was retroreflected by a flat mirror ($R ≈ 99\%$) mounted on a piezoelectric transducer. A neutral-density filter ($T = 50\%$) was used in front of the mirror so that approximately $7.5 \times 10^{-4}$ of the decaying cavity power was reflected back to the cavity. However, the reflected beam was not mode matched to the cavity, since the beam from the cavity was expanding from the cavity waist, whereas the reflection back to the cavity was from a flat mirror. Consequently, the actual fraction of power coupled back into the cavity mode should be...
In summary, we have identified spurious optical reflections as a potential source of noise in cavity ring-down spectroscopy measurements. The effect can be significant: Even a single reflection from a V-layer antireflection coating (R \sim 0.25\%) returned to the cavity used in this Letter can cause a systematic error in the measured absorption of parts in 10^{-8}, far above the random noise limit. Furthermore, air currents and temperature-induced motion will limit the usefulness of averaging multiple cavity decay measurements, since the induced systematic error is interferometrically dependent on the optical path length from the spurious reflector to the cavity. Attention to possible scatter and reflection sources should reduce the systematic deviations of the decay measurements. However, it is likely that the best results will be obtained with an active solution such as the use of piezoelectric transducer–mounted mirrors outside the cavity to modulate the distance to any spurious reflections. The induced error is cyclical and will be randomized by varying the optical path length back to the cavity. In such instances, the variance of each data set is increased, but subsequent averaging results in superior repeatability.

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**References**