SOME IMPLICATIONS OF RECIPROCITY FOR TWO-WAY CLOCK SYNCHRONIZATION*

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ABSTRACT

Two common methods for synchronizing remote clocks are called one-way and two-way. Both of these methods, when operated in the traditional fashion are subject to a number of difficulties related to propagation perturbances. This paper points out however, that under certain circumstances, these difficulties can be circumvented for the two-way scheme. This possibility is explored theoretically, in some detail, with respect to the Loran-C navigation system.

INTRODUCTION

Radio signals are commonly employed to compare clocks at remote locations. The two most commonly used schemes are called "one-way" and "two-way." In the one-way scheme a signal is transmitted from location A, where clock A is located, to location B where clock B is located. The time, τ_{AB}, it takes the signal to travel from A to B depends upon the signal path distance, d, between A and B and upon the average signal speed, s, over the path. Or in simple mathematical terms

$$\tau_{AB} = d/s. \qquad 1)$$

To accurately compare the clocks it is necessary to know τ_{AB}. Although 1) is mathematically simple its determination in the "real world" can be very difficult. There are a number of reasons for this. First, the signal may not travel a "line-of-sight" path between A and B. If the signal, for example, is ionospherically propagated the actual signal path is a complex function of the distribution of electrons in the ionosphere. Second, the signal speed may change along the path. Again, in the ionosphere, the signal speed is a function of electron distribution. Third, the signal may change its shape during propagation. This means that the point on the signal wave form that is "tagged" as the time reference point as the signal leaves A may be "washed out" by the time the signal arrives at B. Fourth, it is necessary to accurately know the geographic locations of A and B.

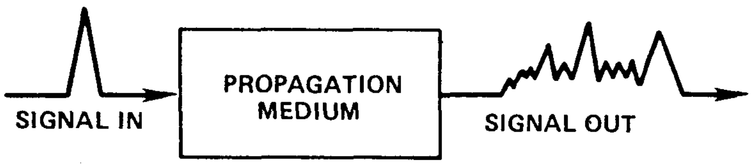
The first three factors are usually discussed in terms of:

1) homogeneity of the medium;
2) isotropy of the medium;

and 3) frequency dependence of the medium (dispersion).

As mentioned earlier the ionosphere is not homogeneous because its electron density changes with height, which leads to non-constant signal speed and to complicated signal paths. Furthermore, the propagation medium may be non-homogenous in the sense that it contains irregularities which scatter the signal. Thus, although only one signal is transmitted from A, several signals may arrive at B via several different paths.

Because of the presence of the earth's magnetic field, the ionosphere is also non-isotropic for radio waves. In general, this means that the signal speed and the attenuation of the signal depend upon direction of propagation. Finally, for radio waves, the ionosphere is frequency dependent because the signal speed depends upon signal frequency. This effect is usually referred to as frequency dispersion of the signal. All three of these factors lead to shape distortion of the signal, illustrated schematically:



Sommerfeld[1] and others have considered dispersion in considerable detail. These treatments are highly mathematical, and I shall only briefly sketch the main results of these investigations. First, some very small part of the signal travels with the speed of light independent of the dispersive properties of the medium. This part of the signal called the "Sommerfeld precursor" is quite weak and oscillates very rapidly. A short time later the "Brillouin precursor" arrives with greater amplitude and longer duration. Finally, depending upon the structure of the transmitted signal and upon the detailed dispersive properties of the medium, the signal settles into some steady state value. The Sommerfeld and Brillouin precursors have been experimentally verified in the laboratory at micro-wave frequencies,[2] although to my knowledge no one has investigated the possibility of using them in timing and navigation systems.

TWO-WAY MEASUREMENTS

To avoid some of the difficulties discussed in previous paragraphs, it is sometimes easier and perhaps even necessary to measure τ_{AB} when one wants to make a clock comparison. Usually a "two-way scheme" is employed to measure τ_{AB} in the following way. As in the one-way method, a signal is transmitted from A which arrives some time later at B. At the instant the signal arrives at B (or after some known delay time), it is returned to A. To determine the round trip path delay between A and B, an observer at A notes the transmission and reception times of the signal at A with respect to the clock at A. If the propagation medium is isotropic, then path delay reciprocity can be assumed; that is, the path delay from A to B equals the path delay from B to A. Thus, τ_{AB} = (round trip delay)/2.

This approach alleviates two problems. First, it is not necessary to know the geographic positions of A and B, and second, it is not necessary to know average signal speed. The disadvantage is that transmitting and receiving equipment are required at both ends of the path. There may also be a problem due to dispersion. If the signal arriving at B is a distorted version of the one transmitted from A, then it is no longer clear when the time reference point has arrived at B so it can be "reflected" to A. Similarily, the signal traveling from B to A will be distorted so there is again the problem of determining signal arrival time at A.

The problem of signal shape distortion can be considered from a somewhat different point of view. As stated above, if reciprocity holds and if there is no signal shape distortion, then the observer at A, using the two-way method, can determine the one-way path delay τ_{AB} from A to B. The concept of signal delay, τ_{AB}, involves the notion of average signal speeds and path distance (as shown explicitly by equation 1). The two-way measurement only provides τ_{AB}, that is, it only provides the ratio of distance d to speed s. If either d or s is known by some independent means then the other quantity can be determined.

Consider the case now where there is some definite known path distance d, say a line of sight path, but there is dispersion along the path so that a distorted signal arrives at B and the return signal also arrives distorted at A. In this case, τ_{AB} cannot be measured, so no meaningful value can be assigned to s, even though d is known. We could say that the notion of signal speed or "group" velocity, as it is usually called, has failed. In a similar fashion suppose that there is not dispersion, but there are many irregularities in the path which scatter the signal so that although only one signal is transmitted from A, many overlapping but similarly shaped signals arrive at B. Again the composite signal at B is a distorted version of the one that left A, so that no meaningful arrival time can be assigned. For this case, s has a definite value (assuming isotropy), but d is not meaningful since no single path is involved. If a single path can be isolated (e.g. the Loran-C ground-wave signal), then the problem can be resolved.

Suppose now that τ_{AB} cannot be meaningfully determined by the two-way method, either because of signal shape distortion or because of a multitude of paths, or perhaps both. Are either one of these conditions sufficient to destroy the utility of the two-way scheme? That is, is the notion of definite path delay, τ_{AB}, and definite average group velocity, s, necessary for the two-way scheme to work?

Let's consider the following situation. The propagation medium between A and B is both dispersive and non-homogeneous, but isotropic. That is, the signals propagating between A and B are both dispersed and scattered identically in both directions because of isotropy. Suppose similary shaped signals are launched simultaneously from A and B. The signals arriving at A and B will have identical shapes, though very different from the transmitted shapes, and further, both signals, since they were launched simultaneously, will fluctuate in amplitude and phase identically as a function of time at A and B. If the signals are not launched simultaneously (and if the propagation medium remains constant with time), then the two signals arriving at A and B will still be identically shaped, but displaced in arrival time by an amount that is just equal to the difference in launch times of the two signals.

Thus, all that is required to compare the clocks at A and B is to determine the amount of time displacement of the two signals in spite of the fact that the notions of group velocity and definite path delay have no meaning. Thus, isotropy (with the two-way scheme) is the only condition required to compare clocks. Homogeneity and dispersionless media are not required.

This fact does not seem to have been explicitly pointed out before, perhaps because of the intimate association between timing and navigation systems where the notion of path delay is critical.

In summary then, if the medium is despersionless, isotropic and homogeneous, the notion of path delay can be employed and the two-way scheme may be employed in the usual way. However, if the medium is dispersive and non-homogeneous, the two-way scheme can still be used if the received signals at the two ends of the path are brought together to determine their difference in arrival time. In fact, we might say that bringing the records together is the extra price we must pay to remove the dispersion and non-homogeneity problems.

A practical implementation of this procedure would be to sample, at high rate, and store on magnetic tape, the amplitudes of the two received signals as a function of time with respect to the clocks at A and B. The tapes could then be brought together and lag cross-correlated to determine the clock offsets.

LORAN-C

Loran-C is the backbone of the system for international clock comparisons. Loran-C has the advantage that its signals are pulsed so that ground wave and sky-wave signals can be separated if the observer is sufficiently close to the Loran-C transmitter. However, at distances beyond several thousand kilometers, the ground wave weakens relative to the sky wave signal and the difference in arrival time between the sky and ground wave signals becomes small so that it is difficult to separate them. Even at distances where the separation can be made, international clock comparisons are compromised by the fact that the ground wave delay is subject to an annual variation with a magnitude of about one microsecond at sites as far removed as the NBS time scale in Boulder, Colorado.[3]

The discussion in this paper suggests that more accurate clock comparisons could be made if Loran-C were employed in a two-way mode. First of all, variations in path delay (annual or otherwise) cancel out. Second, it is not necessary to separate the ground and sky waves if the cross-correlation technique is utilized. Third, the Loran-C sky wave has been detected at distances exceeding

5 thousand kilometers,[4] so it would not be necessary to "bridge" large distances by intercomparing observations of Loran-C signals which were all within "groundwave" distance of each other. Fourth, to improve signal to noise, the signals could be averaged for long periods of time at both ends of the path, since signal path delay variations have no effect on the cross-correlation determination of clock offset.

Strictly speaking, for the two-way measurements, the observers at both ends of the path should be co-located with the transmitting antennas at A and B, but as a practical matter, this is not possible. However, other measurements[5] suggest that the observer could be as far as a few kilometers from the transmitting antenna before any significant difference forward and return in the propagation paths developed. Another difficulty related to being near the transmitter antennas, is that the transmitter signals might interfere with one's ability to receive distant Loran-C signals. However, because of the short pulse width of the signals, it appears[6] that gating procedures can be developed which will solve this problem.

The primary point that remains in question is the degree of an isotropy for Loran-C sky-wave signals. As stated earlier, the presence of the earth's magnetic field in the ionosphere makes it anisoptropic. Using a procedure developed by Johler,[7] some preliminary calculations have been made to determine the degree of anisotropy for Loran-C sky waves. Table I shows the results of these calculations for both local noon and local midnight at the mid-point of the path. When the observer is far enough from the transmitter station so that the signals reflect from the ionosphere at grazing incidence, i.e., at or exceeding 2000 kilometers, the table shows during the daytime that the path delay non-reciprocity at 100 kHz is 49 nanoseconds for east-west propagation and 3 nanoseconds for north-south propagation. At night, the non-reciprocity amounts to 190 nanoseconds for east-west propagation. Other related calculations[8] suggest that one can always expect a greater non-reciprocity at night.

Based on these results for grazing incidence, if the non-reciprocity component of the 100 kHz signal delay is ignored, the error in the two-way clock comparison would be half the total non-reciprocity or about 25 nanoseconds.

Of course these calculations depend upon a particular model of the ionosphere. However, as long as the signals reflect from the ionosphere at grazing incidence, I do not anticpate that the degree of non-reciprocity will be particularly sensitive to the details of the ionospheric model.[9]

The table also shows that as the observer's distance to the Loran-C transmitter decreases the degree of non-reciprocity increases. This is probably due to the fact that, at shorter distances, the signal penetrates more deeply into the ionosphere during the reflection process, so that the signal path through the non-isotropic portion of the total path between A and B increases. My preliminary conclusion from these calculations is that two-way Loran-C comparisons should be made during the daytime over distances large enough for grazing incidence to hold.

It should be emphasized that these calculations apply only to the 100 kHz Fourier component of the Loran-C pulse. To determine in detail what happens to the entire pulse during propagation, requires making similar calculations for all of the significant

Fourier components of the pulse and then adding up these components with the proper phases at the observer's location. In addition, the degree of attenuation of the amplitudes of the Fourier components during propagation is also a function of Fourier frequency and propagation directions. Therefore, a complete analysis of what happens to the Loran-C pulse during propagation must take into account both amplitude and phase delay variations as a function of direction and Fourier frequency.

The advantage of such an analysis is that the full energy in the pulse at all Fourier components can be used. Such calculations are now under way and will be reported in a later paper.

As a final point, since the procedures discussed here imply that clocks comparisons in the tens of nanoseconds range be accomplished, relativity effects cannot be ignored. For example, in the east-west direction at 40° Lat., over a distance of about 4000 kilometers, non-reciprocity due to relativistic effects amounts to about 10 nanoseconds.[10]

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TABLE I

THEORETICAL CALCULATIONS OF NON–RECIPROCITY

DISTANCE KILOMETERS	DIRECTION	FREQUENCY	LOCAL TIME OF DAY AT MID-POINT OF PATH HOURS	NON-RECIPROCITY NANOSECONDS
2000	EAST-WEST	100 kHz	0	190
"	"	"	12	49
1500	"	"	0	285
1000	"	"	0	360
"	"	"	12	67
800	"	"	0	458
"	"	"	12	86
700	"	"	0	427
"	"	"	12	129
600	"	"	12	343
2000	NORTH-SOUTH	"	12	3