

# TOTAL HADAMARD VARIANCE: APPLICATION TO CLOCK STEERING BY KALMAN FILTERING

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**Abstract**—The Total variance approach has been developed for increasing the confidence of the estimation of the classical Allan variance, particularly for large integration times. This method is based on a procedure involving extension of the original data. Recently, we showed how this approach may be applied to all classes of structure functions (i. e. variances) relevant to the time and frequency community.

In particular, we obtained an improvement in the confidence of the estimation of the Hadamard variance. The utility of this variance is its insensitivity to linear frequency drifts and its convergence for very low frequency noises ( $f^{-4}$  FM). As a consequence, the Hadamard variance is often used for estimating low frequency noises without significant influence from drift.

As an example, this variance has a primary use in GPS and its master control operations. Parameters of the Hadamard function are used to estimate  $q$  coefficients in the GPS Kalman algorithm. In this paper, we propose applying this method to clock steering in a more general context. The total approach increases the equivalent degrees of freedom (edf) of the  $q$  estimates. We give simple formulae for computing the edf and removing the bias induced by the total approach.

## 1. INTRODUCTION

Using a type of Hadamard variance, the goal of this paper is to reduce the uncertainty of long-term estimates of frequency stability without increasing the length of a data run. For measurements of frequency stability, the two-sample frequency variance known as the Allan variance was generalized to an  $N$ -sample variance weighted with binomial coefficients by R. A. Baugh (Ref. 1). The case of the three-sample frequency variance that is used here is the Picinbono variance (Ref. 2) times  $\frac{3}{2}$ . However, in this paper, it will be called a Hadamard variance (following Baugh's work) that is defined as follows. Given a finite sequence of frequency deviates  $\{y_n, n = 1, \dots, N_{ymax}\}$ , presumed to be the measured part of a longer noise sequence and with a sampling period between adjacent observations given by  $\tau_0$ , define the  $\tau = m\tau_0$ -average frequency deviate as

$$\bar{y}_n(m) \equiv \frac{1}{m} \sum_{j=0}^{m-1} y_{n+j}. \quad (1)$$

Let  $H_n(m) = \bar{y}_n(m) - 2\bar{y}_{n+m}(m) + \bar{y}_{n+2m}(m)$  be the second difference of the time-averaged frequencies over three successive and adjacent time intervals of length  $\tau$ .

Define the Hadamard variance as

$${}_H\sigma_y^2(\tau) = \frac{1}{6} \langle H_n^2(m) \rangle, \quad (2)$$

where  $\langle \cdot \rangle$  denotes an infinite time average over  $n$ , and  ${}_H\sigma_y^2$  depends on  $m$ .

The GPS program office uses this particular time-series statistic for estimating Kalman algorithm coefficients according to (Ref. 3), which coefficients will be discussed in a later section. The Hadamard deviation  ${}_H\sigma_y(\tau)$  is a function that can be interpreted like the more efficient Allan deviation as a frequency instability *vs.* averaging time  $\tau$  for a range of frequency noises that cause different slopes on  ${}_H\sigma_y(\tau)$ . This is shown in figure 1. For estimating Kalman drift noise coefficients,  ${}_H\sigma_y(\tau)$  is inherently insensitive to linear frequency drift and reports a residual “noise on drift” as a  $\tau^{\frac{3}{2}}$  slope, or what is commonly called random run FM (RRFM). This is in contrast to the Allan deviation, which is sensitive to drift and causes a  $\tau^{+1}$  slope. If the level of drift is relatively high, it masks the underlying random noise. It is customary to estimate and remove overall frequency drift. Depending on the method of drift removal, this procedure can significantly alter the Allan deviation in the longest term  $\tau$  region of interest, so estimating underlying noise can be a formidable task for any given data span. On the other hand, the Hadamard deviation is unaffected by removing overall frequency drift. For this reason, it is the preferred statistic in situations in which the frequency drift may be above the random noise effects, which is the case with the use of Rb clocks in the GPS Block II satellite program. We do not imply that systematics such as frequency drift can be ignored. Indeed, satellite clocks are changed and these systematics must be learned as quickly as possible to ensure a smooth changeover.

Throughout this writing, we will make comparisons using the traditional best statistical estimators, denoted by “Hvar” and “Avar” referring to the maximum-overlap estimators of the Hadamard and Allan variances. Section 2 reviews the “total” approach to improving statistical estimation. Sections 3 and 4 give two methods of computing total Hadamard variance, designated as TotHvar, using measurements first of fractional frequency deviations

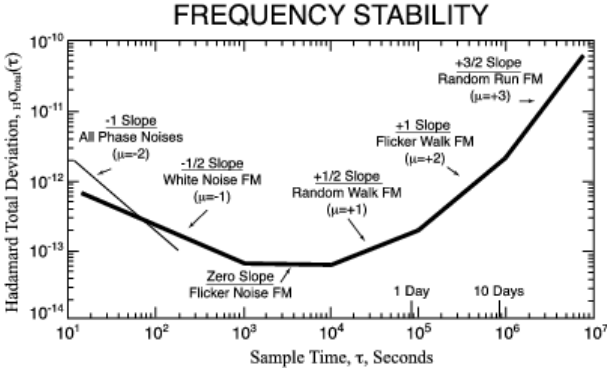


Fig. 1. The Hadamard deviation (root Hvar) shows FM power-law noises as straight lines in addition to PM sources of noise for  $\tau$ -domain power-law exponent  $\mu$  (that is,  $\text{H}\sigma_y^2(\tau) \propto \tau^\mu$ ) range of  $-2 \leq \mu \leq 3$ . We define a new estimator that can be interpreted identically called Hadamard-total deviation (root TotHvar) and that has significantly improved confidence at long term. The Hadamard-total deviation is insensitive to linear frequency drift that can mask characteristic random noise typically encountered here in the region where  $\tau =$  one-week and longer. The goal is to identify  $\mu$  even-integer power-law noises and accurately estimate their levels in order to set system parameters associated with the GPS Kalman filter.

and then of time deviations. Then we quantify the advantage of TotHvar over Hvar in Section 5, giving formulae for computing bias and equivalent degrees of freedom (edf) of TotHvar. Section 6 reviews how an estimate of  $\tau$ -domain frequency stability is used to set Kalman filter parameters (or  $q$ 's) used in GPS operations. Finally, Section 7 discusses the application of the total Hadamard variance in the more general context of clock steering.

## 2. THE “TOTAL” APPROACH

The total estimator approach has been developed to improve confidence of major statistical tools used in analyzing and characterizing instabilities in phase and frequency of oscillators and synchronization systems (Refs 4, 5, 6, 7, 8, 9, 10). Making a “total” estimator of eqn. (2) involves joining each real data subsequence, namely the subsequence of  $y_i$  that goes into each  $H_n(m)$  term, at both its endpoints by the same original data subsequence so that it repeats. This creates a new extended version of each  $y_i$  subsequence that may be extended by a forward or backward repetition, with or without sign inversion, thus with four possible ways to extend. From numerous simulation studies, we have determined that an extension by even (uninverted) mirror reflection of linear-frequency-detrended  $H_n(m)$  subsequences yields the largest edf gain and least bias for the range of noise types identified by standard Hvar. This is described in the next section.

## 3. COMPUTATION USING $y_n$ -SERIES

$H_n(m)$  is computed from a  $3m$ -point data segment or subsequence  $\{y_i\}_n = \{y_i, i = n, \dots, n+3m-1\}$ . Before applying any data extensions, we must remove a linear

frequency trend (drift) from each subsequence by making

$${}^\circ y_i = y_i - c_1 i,$$

where  $c_1$  is a frequency offset that is removed to minimize  $\sum_{i=n}^{n+3m-1} ({}^\circ y_i - \overline{{}^\circ y_i})^2$ , to satisfy a least-squared-error criterion for the subsequence. In practice, it is sufficient to compute this background linear frequency slope by averaging the first and last halves of the subsequence divided by half the interval and subsequently subtracting the value. Now extend the “drift-removed” subsequence  $\{{}^\circ y_i\}_n$  at both ends by an uninverted, even reflection. Utility index  $l$  serves to construct the extensions as follows. For  $1 \leq l \leq 3m$ , let

$${}^\circ y_{n-l}^\# = {}^\circ y_{n+l-1}; \quad {}^\circ y_{n+3m+l-1}^\# = {}^\circ y_{n+3m-l}; \quad (3)$$

to form a new data subsequence denoted as  $\{{}^\circ y_i^\#\}_n$  consisting of the drift-removed data in its center portion, plus the two extensions, and thus having a tripled range of  $n-3m \leq i \leq n+6m-1$  with  $9m$  points. To be clear, we now have *extended subsequence*  $\{{}^\circ y_i^\#\}_n = \{{}^\circ y_i^\#, i = n-3m, \dots, n+6m-1\}$ . Define

$$\begin{aligned} \text{Total}_H \sigma_y^2(m, \tau_0, N_{y_{max}}) &= \frac{1}{6(N_{y_{max}} - 3m + 1)} \\ &\times \sum_{n=1}^{N_{y_{max}} - 3m + 1} \left( \frac{1}{6m} \sum_{i=n-3m}^{n+3m-1} ({}^\circ H_i^\#(m))^2 \right), \end{aligned} \quad (4)$$

for  $1 \leq m \leq \lfloor \frac{N_{y_{max}}}{3} \rfloor$ , where  $\lfloor c \rfloor$  means the integer part of  $c$  and notation  ${}^\circ H_i^\#(m)$  means that  $H_n(m)$  above is derived from the new triply-extended subsequence  $\{{}^\circ y_i^\#\}$ . The symmetries of the extension and the Hvar filter allow the computational effort to be halved as follows. Let  $k = \lfloor 3m/2 \rfloor$ . We need to calculate  $\#y_i^\circ$  only for  $n-k \leq i \leq n+k+3m-1$ , and  $\#H_i^\circ(m)$  only for  $n-k \leq i \leq n+k$ . Then

$$\begin{aligned} \sum_{i=n-3m}^{n+3m-1} (\#H_i^\circ(m))^2 &= 2 \sum_{i=n-k+1}^{n+k-1} (\#H_i^\circ(m))^2 \\ &\quad + (\#H_{n-k}^\circ(m))^2 \\ &\quad + (\#H_{n+k}^\circ(m))^2, \quad m \text{ even}, \\ &= 2 \sum_{i=n-k}^{n+k} (\#H_i^\circ(m))^2, \quad m \text{ odd}. \end{aligned} \quad (5)$$

## 4. COMPUTATION USING $x_n$ -SERIES

The methodology described above can be written in terms of calculations on residual time differences between clocks, namely an  $x_i$ -series (to adhere to usual notation), recalling that

$$\bar{y}_i(m) = (x_{i+m} - x_i) / (m\tau_0).$$

Thus in the total approach applied to  $x_i$ -series, the data extensions on subsequences of  $x_i$  will be constructed in such a way that

$${}^\circ y_i^\# = \left( {}^\circ x_{i+1}^\# - {}^\circ x_i^\# \right) / \tau_0,$$

in agreement with section 3 above. This has the effect of requiring an odd mirror extension and a third-difference operator when considering subsequences of  $x_i$ . The Hadamard variance discussed in section 3 as a second-difference operator on  $\tau$ -averaged  $y_n$  values can now be re-expressed in terms of a third-difference operator on time-error  $x_i$ -values. The sample variance (or mean square) of these third differences falls neatly into a class of structure functions, namely the variance produced by a difference operator of order three (Ref. 10). The modified Allan variance can also be treated as a third-difference variance (Ref. 11).

The  $x_i$ -subsequence that corresponds to the  $y_i$ -subsequence starting at  $n$  is  $\{x_i, n \leq i \leq n+3m\}$ , which has  $3m+1$  terms. Compute the detrended subsequence  ${}^\circ x_i$  according to

$$k = \left\lfloor \frac{3m}{2} \right\rfloor, \quad c_2 = \frac{x_n - x_{n+k} - x_{n+3m-k} + x_{n+3m}}{k(3m-k)},$$

$${}^\circ x_i = x_i - \frac{1}{2} c_2 (i-n)(i-n-3m), \quad n \leq i \leq n+3m.$$

Define the extended subsequence  $\left\{ {}^\circ x_i^\#, n-3m \leq i \leq n+6m \right\}$  by

$$\begin{aligned} {}^\circ x_i^\# &= {}^\circ x_i, \quad n \leq i \leq n+3m, \\ {}^\circ x_{n-l}^\# &= 2({}^\circ x_n) - {}^\circ x_{n+l}, \quad 1 \leq l \leq 3m, \\ {}^\circ x_{n+3m+l}^\# &= 2({}^\circ x_{n+3m}) - {}^\circ x_{n+3m-l}, \quad 1 \leq l \leq 3m. \end{aligned}$$

Then

$$m\tau_0 \left( {}^\circ H_i^\#(m) \right) = -{}^\circ x_i^\# + 3 \left( {}^\circ x_{i+m}^\# \right) - 3 \left( {}^\circ x_{i+2m}^\# \right) + {}^\circ x_{i+3m}^\#$$

$$n-3m \leq i \leq n+3m-1,$$

where  ${}^\circ H_i^\#(m)$  has the same meaning as in Section 3. Now the Hadamard-total variance is computed from (4) as before with  $N_{y_{max}} = N_{x_{max}} - 1$ . Because of symmetry we need  ${}^\circ x_i^\#$  only for  $n-k \leq i \leq n+k+3m$ , and (5) applies.

## 5. BIAS AND EQUIVALENT DEGREES OF FREEDOM

We consider the random frequency-modulation (FM) noises since these dominate at long-term averaging times where we can capitalize on the improved confidence of using the total approach. To analyze phase-modulation (PM) noises, one would usually use Total TDEV (Ref. 6) rather than the Hadamard deviation. For brevity, let  $\text{Total}_H \sigma_y^2(m, \tau_0, N_{y_{max}})$  be  $\text{TotHvar}(\tau, T)$ , where  $\tau =$

TABLE I  
COEFFICIENTS FOR COMPUTING (6) AND (7), NORMALIZED BIAS  
AND EDF OF TOTHVAR.

Noise	Abbrev.	$\alpha$	$a$	$b_0$	$b_1$
White FM	WHFM	0	-0.005	0.559	1.004
Flicker FM	FLFM	-1	-0.149	0.868	1.140
Random Walk FM	RWFM	-2	-0.229	0.938	1.696
Flicker Walk FM	FWFM	-3	-0.283	0.974	2.554
Random Run FM	RRFM	-4	-0.321	1.276	3.149

$m\tau_0, T = N_{y_{max}}\tau_0$ . The normalized bias and edf for TotHvar are given by

$$\text{nbias}(\tau) = \left[ \frac{E\{\text{TotHvar}(\tau, T)\}}{E\{\text{Hvar}(\tau, T)\}} - 1 \right] = a, \quad (6)$$

$$\text{edf}(\tau) = \text{edf}[\text{TotHvar}(\tau, T)] = \frac{T/\tau}{b_0 + b_1\tau/T}, \quad (7)$$

where  $E\{\cdot\}$  is expectation of  $\{\cdot\}$ ,  $0 < \tau \leq \frac{T}{3}$ ,  $\tau \geq 16\tau_0$  (to be explained), and  $a$ ,  $b_0$ , and  $b_1$  are given in Table I for the five FM noise types considered by the Hadamard variance.  $\alpha$  is the corresponding power-law exponent of the fractional-frequency noise spectrum  $S_y(f) \propto f^\alpha$ . In the context here, its valid range is  $-4 \leq \alpha \leq 2$ .  $E\{\text{TotHvar}(\tau, T)\}$  relative to  $E\{\text{Hvar}(\tau, T)\}$  in (6) is independent of  $\tau$  and  $T$ , dependent on noise type, and biased low, giving  $a$  the negative sign in Table I. The edf formula (7) is a convenient, empirical or "fitted" approximation with an observed error below 10% of numerically computed exact values derived from Monte-Carlo simulation method using the  $b_0$  and  $b_1$  coefficients of Table I and with the error decreasing with averaging factor  $m = \tau/\tau_0$  increasing.

To show the improvement in estimating the Hadamard function, Table II lists the exact values of edf from theory for computations using TotHvar *vs.* plain Hvar for the longest averaging factor in which  $\tau = T/3$ . This point is the last point in the estimate, and the improvement in confidence using TotHvar is substantial, particularly for the general case of WHFM noise. TotHvar is a significantly improved estimator that offsets much of the criticized inefficiency in using the sample Hadamard deviation as opposed to the sample Allan deviation in the presence of common WHFM noise in frequency standards.

## 6. THE KALMAN NOISE MODEL AND THE GPS OPERATIONS PROBLEM

The time update of clock states in the Master Control Station (MCS) Kalman prediction algorithm is based on an average of the the most recent measurement of these states for each individual clock, modeled simply by random noise acting on phase  $x(t)$ , frequency  $y(t)$ , and frequency drift  $z(t)$ . With this model, the measured power-

TABLE II  
EXACT  $\frac{\text{edf}\{\text{TotHvar}(T/3, T)\}}{\text{edf}\{\text{Hvar}(T/3, T)\}}$  GAIN FOR  $\tau_{max} = T/3$ .

Noise	edf gain of TotHvar( $T/3, T$ )
WHFM	3.447
FLFM	2.448
RWFM	2.044
FWFM	1.676
RRFM	1.313

law  $\alpha$  exponents of the frequency-fluctuation noise spectrum take on only the values 0, -2, and -4, corresponding to WHFM, RWFM, and RRFM, or  $\mu = -1, 1, \text{ and } 3$  in the  $\tau$ -domain. Hence, we want to precisely extract the level of these noises for each clock using the most efficient method possible, which heretofore has been the sample Allan variance with drift removed from the data run, and more recently the sample Hadamard variance, because of its logical link to the model. If white PM (WHPM) is a significant noise component, and for completeness, the  $\alpha = 2, \mu = -2$  case corresponding to WHPM is included as a separate error.

The parameters used by the MCS within GPS system operations are denoted as Kalman filter  $q$ 's. By convention, each filter parameter  $q_i, i = 0, 1, 2, 3$  corresponds respectively to  $\tau$ -domain power law exponents  $\mu = -2, -1, 1, 3$ . For the Hadamard variance, the relationship is (Ref. 3)

$$H\sigma_y^2(\tau) = \sigma_{WHPM}^2 + \sigma_{WHFM}^2 + \sigma_{RWFM}^2 + \sigma_{RRFM}^2 \\ = \frac{10}{3}q_0\tau^{-2} + q_1\tau^{-1} + \frac{1}{6}q_2\tau + \frac{11}{120}q_3\tau^3. \quad (8)$$

Tuning the Kalman filter depends on the ability to “ $q$ ” each individual clock according to estimates of its noise. The GPS Block IIR satellite program incorporates Rb atomic oscillators that are characterized by a mix of various levels and types of random noise and with frequency drift that may be significantly above noise. Using “frequency-drift insensitive” Hvar and using (8), the confidence becomes a factor of about  $\frac{1}{3}$  less than using Avar near the last and crucial long-term  $\tau_{max} = T/3$  value. This is because the plain sample Hadamard's edf is one less than Allan's edf. For the proper perspective, note that we are in the one-week averaging  $\tau$ -region with a last real-time data run of about one month, thus  $\text{edf} \approx 1-2$ ; so estimating filter  $q$ 's is somewhat subjective. Figure 2 illustrates a summary of estimates of frequency stability for each GPS satellite clock as published in reports issued by the Naval Research Laboratory (Ref. 13).

Table II shows that the new TotHvar( $T/3, T$ ) edf is multiplied by a factor of 1.3 to 3.4 over plain Hvar( $T/3, T$ ). TotHvar can be applied directly and reliably, while retaining the efficiency of the sample Allan variance without the difficulty associated with real-time drift removal.

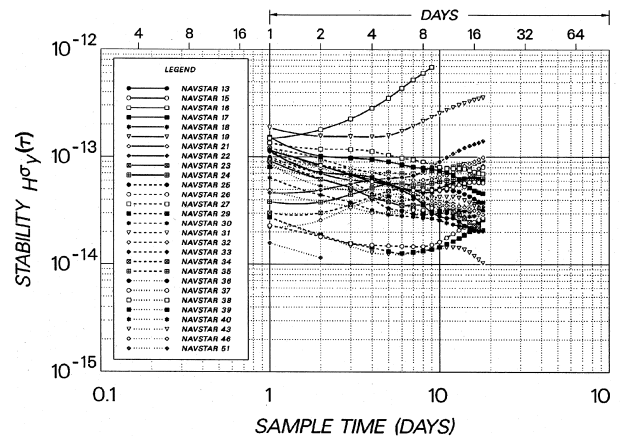


Fig. 2. Hadamard-deviation frequency stability of individual GPS satellite clocks *vs.* USNO Master Clock for the period 1 January to 1 July, 2000 (Ref. 18).

The work of this paper has impact on two GPS operational issues. The first is that the time needed to estimate the Hadamard variance is substantially reduced. For example, to obtain a  $\tau =$  one-week estimate of the Hadamard variance with, say, the last 40 days of measured data, the Total approach using TotHvar obtains a one-week estimate with the same or better confidence in about 26 to 34 days of measured data (see figure 3). The second issue is that satellite data are obtained by the linked common-view method (Ref. 14), and the delay in receiving the monitor station tracking data is currently at 2 to 3 days. Thus, it is important to extract maximum information from data at hand.

## 7. APPLICATION TO CLOCK STEERING

In many applications, the time given by an oscillator must be modeled and predicted. As a consequence, the time prediction performance of this clock has to be characterized.

In order to estimate the time uncertainty given by an oscillator, a linear or parabolic fit may be performed over a sequence of observed time deviations and extrapolated during a prediction period. Thus, the requirements of synchronization are specified by the maximum error of the time deviation prediction from the extrapolated fit.

The question is then, “How is this maximum error related to the noise levels of the clock?” Let us call Time Interval Error (TIE) the differences between the extrapolated parabola and the real time deviation. The TIE is due to two effects: (1) the error of determination of the parabolic parameters and (2) the error due the noise of the clock. Obviously, both of these errors may be positive or negative, and the ensemble average of the TIE is equal to zero. Moreover, it can be easily shown that the distribution in TIE is Gaussian. Consequently, we only need to estimate the variance of the TIE in order to define its statistical characteristics.

The theoretical variance of the TIE versus the noise

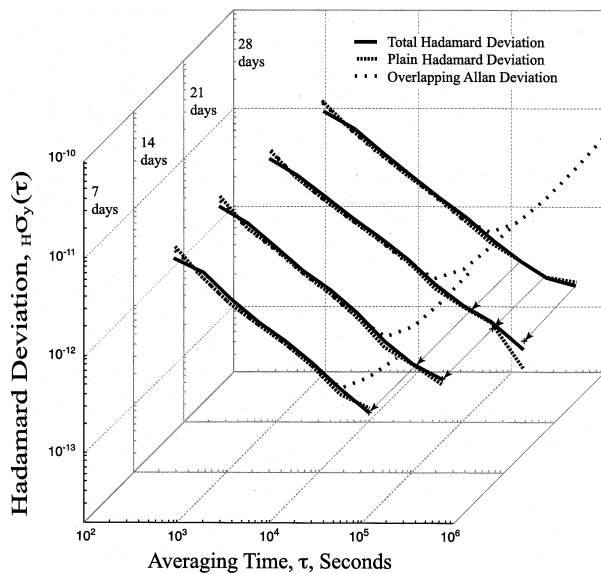


Fig. 3. Total Hadamard deviation, plain Hadamard deviation, and Allan deviation for SV24 satellite clock data as the data run increases from 7 days (front plot) to 28 days (rear plot). The last (rightmost) values of TotHdev for shorter data runs anticipates the underlying noise level of longer runs compared to plain Hdev (arrowed lines are projected off 28-day data run). The Allan deviation's response to frequency drift masks the long-term noise level.

levels has been calculated in Ref. 15 which shows that the time prediction performance is directly linked to the accuracy of the noise level estimation. The estimates of the TIE standard deviation are distributed following a Student law, and the edf of these estimates are the same as the edf of the dominating noise level. If the time deviation sequence contains a parabola with an amplitude that is much higher than the random fluctuations, one must use a noise level estimator which is insensitive to quadratic drift while still sensitive to low frequency random noise. The Total Hadamard variance described in this paper is the most reliable estimator for such applications.

## 8. CONCLUSION

We have developed a significantly improved estimator of the three-sample Hadamard frequency variance based on the so-called "total" approach and denoted as TotHvar, for use in GPS operations and analysis. Practically speaking, we have reduced the long-term estimation uncertainty in terms of edf by a factor of 1.3 to 3.4, depending on the noise type. Having confidence greater than plain Hvar and even equal to or greater than Avar, TotHvar is a statistic that permits tuning of the MCS Kalman filter with more accurately chosen clock-estimation parameters (or  $q$ 's) that are linked to the most recent measurements of frequency stability of each clock. The increased confidence from TotHvar and shorter data processing delays will play significant roles in adequately managing future GPS system events. More generally, the Total Hadamard variance may be used in any application

needing an accurate estimation of the low frequency noise levels when a high level of frequency drift is present in the signal.

## 9. ACKNOWLEDGEMENTS

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