LASER TO MICROWAVE FREQUENCY DIVISION USING SYNCHROTRON RADIATION II

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Abstract

We present a review of theoretical calculations which demonstrate the feasiblity of obtaining one step frequency division from optical or infrared laser frequencies to a subharmonic in the microwave spectral region, and include current experimental designs toward a practical realization of this goal. We plan to drive the cyclotron orbit of a single relativistic electron, which is confined in a Penning ion trap, with a laser beam focused to a spot diameter $\sim \lambda$. This method is an extension of a common technique used in cyclotrons and synchrotrons where the orbit of high energy particles is driven at a harmonic of the orbit frequency. Our experiment is designed to measure this orbit frequency which is then a subharmonic of the driving (laser) frequency. This technique requires that the uncertainty in the electron orbit dimensions be limited to $\lesssim \lambda/2$, which is possible by radiative cooling and the method of motional sideband excitation. The possibility of a unified optical wavelength/frequency standard is evident.

Introduction

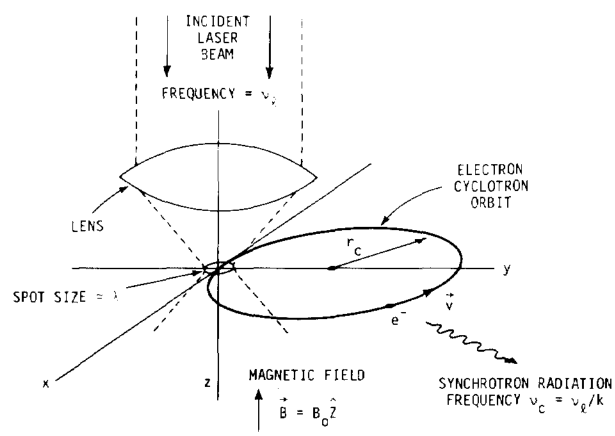
It is our intent in this paper to summarize and extend the theoretical analysis of a broadband laser to microwave frequency divider proposed earlier.[1] We also give a brief description of experimental designs being considered for the realization of this division. The importance of accurate freuqency division from the optical spectrum derives primarily from frequency (time) and wavelength metrology and the extreme likelihood that the most accurate, reproducible, and stable oscillators may be realized in that part of the electromagnetic spectrum. Certainly, there is great value of such a device in the area of atomic and molecular spectroscopy.

Principle of Operation

The proposed synchrotron divider is based on the principle that a purely harmonic oscillator can be coherently driven (pumped) by a spatially nonuniform, higher order harmonic driving field. This method has been used for years in cyclotrons, synchrocyclotrons, and synchrotrons to accelerate charged particles to relativistic speeds. In these devices, particles are frequently driven by radiation from microwave cavities, which is localized to a small portion of the cyclotron orbit, at frequencies integrally related to the cyclotron orbit frequency. For a significant transfer of energy from the RF driving field into the cyclotron orbit it is necessary that the interaction time of the charged particles with the radiation field be comparable to or less than one-half period of the RF frequency. Longer interaction times quickly average the transfer of energy to zero.

It would seem possible to straightforwardly extend this technique to driving fields of shorter wavelengths in order to drive the cyclotron orbit of a single electron in a magnetic field at a very high harmonic of the cyclotron frequency. Theoretically, this is the case, however, shorter wavelengths entail special practical considerations discussed below. If the orbit is stable then the power absorbed by the electron from the harmonic excitation is balanced by the emitted synchrotron radiation. The cyclotron frequency is then a phaselocked submultiple of the driving frequency.

We illustrate the important features of the proposed method in fig. 1.



SYNCHROTRON FREQUENCY DIVIDER

Radiation from a well collimated, Gaussian laser beam, linearly polarized in the x direction and traveling in the -z direction is focused into a spot with diameter $\sim \lambda$ by a lens. If it is possible to provide that the electron orbit, which is confined to lie in the x,y plane, pass through this focus, then the electron will absorb energy from the radiation field. Assuming that the envelope for the electric field vector is approximately Gaussian and that the absorbed energy per pass (ΔW) is small compared to the total electron energy, we have

$$\Delta W \cong \int_{-\infty}^{\infty} eE_x dx = \sqrt{2}\ eA \cos \delta \exp \left\{ -\left[\frac{\pi c S_o}{2v\lambda} \right]^2 \right\} \tag{1}$$

where

A = field amplitude factor
δ = phase of laser field
v = electron's velocity
S_o = actual diameter of focused line ($\sim \lambda$)
c = speed of light

Note that ΔW shows a double exponential dependence on the ratio $cS_o/\lambda v$. Thus the energy transferred to the electron is extremely small unless this ratio is approximately equal to one. This is because the oscillating driving field will rapidly average the energy transfer to zero unless we can arrange that the electron spend approximately one cycle or less in the radiation field. If the extent of the interaction region could be made substantially less than λ then the speed of the electron could be made correspondingly less while maintaining an effective energy transfer. A possibility may be a dielectric waveguide with transverse dimensions less than λ that is projected into the trap and is cut off within about λ of the electron orbit.

Confinement of the circular electron orbit path to $\lesssim \lambda/2$, which is necessary to prevent $\Delta W \to 0$, is accomplished by trapping the electron in a Penning trap with hyperbolic electrodes immersed in a static magnetic field $\vec{B} = B_o\,\hat{z}$. The electrostatic potential is given by $\psi = V_o(r^2-2(z^2-z_o^2))/(r_o^2 + 2z_o^2)$ where r is the radial coordinate, z is the axial coordinate, r_o and z_o are the characteristic dimensions of the trap,[2] and V_o the voltage applied between the trap electrodes. The axially symmetric, static electromagnetic trap is thus formed with two endcap electrodes which are constructed to lie on the $\psi = 0$ equipotential surface and a ring electrode which conforms to the equipotential $\psi = V_o$. The motion of the electron is given by the synthesis of three separate oscillations. First there is the harmonic motion parallel to the z axis at frequency $\nu_z = (eV_o/m\pi^2(r_o^2 + 2z_o^2))^{1/2}$ due solely to the applied electric field. In the radial plane the motion is comprised of the sum of two vectors $\vec{r}_c$ and $\vec{r}_m$ which rotate at frequencies ν_c' and ν_m. These frequencies are approximated by $\nu_m \cong \nu_z^2/(2\nu_c)$ and $\nu_c' \cong \nu_c = (eB_o/2\pi mc)$, when $\nu_c >> \nu_z$. In order to satisfy our requirement to confine the electron orbit path to $\leq \lambda/2$, $|\vec{r}_m|$ and z must be held to $\leq \lambda/4$. This can be accomplished by radiatively cooling the axial (z) motion and suppressing the magnetron (r_m) motion by the method of motional sideband excitation.[1,3] The electron orbit is then nearly circular with cyclotron frequency ν_c'.

The minimum r_m is given by[1]

$$r_m^2 \cong 2\ (\omega_m/\omega_z)\ \langle z^2 \rangle \tag{2}$$

where $\langle z^2 \rangle$ is the mean square thermally excited axial amplitude. As a quantitative example, assume that $r_o \cong 1.6\ z_o = 0.1$ cm, $V_o = 10$ kV, $v/c = 0.8$, $S_o = \lambda$, $B_o \cong 1$T with $\nu_c \sim 15$ GHz, and $T_z = 4$ K. These conditions will give $\nu_z \cong 7.8$ GHz, $\sqrt{\langle z^2 \rangle} \cong 175$ nm, and $r_m \sim 150$ nm and an orbit confined to less than $\lambda/2$ provided $\lambda \gtrsim 700$ nm. We note that V_o is extremely large in order to confine the axial z excursions to $< \lambda/4$. This requirement is greatly reduced if the focus is a cylindrical line parallel to z rather than a spot of dimensions λ on all sides. This focus ($\lambda \times \lambda \times 20\lambda$) could be obtained with a softly focused beam followed by a strong focusing cylindrical lens. A smaller value of V_o reduces the likelihood of spurious field emission currents which could give rise to extraneous trapped electrons. Since $\langle z^2 \rangle \propto V_o^{-1}$ and $\nu_z \propto V_o^{1/2}$, a reduction of V_o by 10^2 increases $\sqrt{\langle z^2 \rangle}$ by 10 and reduces ν_z by the same factor. From Eq. 2, the minimum value of r_m increases by only a factor of three. Thus, this change in V_o to 100 V, allowed by the cylindrical lens focus, gives $\nu_z \cong 780$ MHz, $\sqrt{\langle z^2 \rangle} \sim 1.75\ \mu$m, and $r_m = 0.45\ \mu$m, permitting $\lambda \gtrsim 600$ nm. (It is further noted that the electron orbit confinement problem is greatly reduced by choosing longer wavelength lasers).

If we now provide an energy balance between the energy absorbed by the electron and the energy lost through synchrotron radiation, which is efficiently coupled out to the frequency measurement electronics, the the cyclotron frequency will phase lock to a subharmonic of the laser frequency. Given that the laser has only minimum amplitude and frequency stability,[1] the phase lock condition will occur for $0<\delta<\pi/2$. Consider that $\delta = \pi/4$ initially and that the laser power incident on the cylindrical lens is approximately 20 mW. The power absorbed and subsequently radiated by the electron, confined as described above, is about 1.4×10^{-10} W.

Feedback

Stability of the phase lock is automatically achieved since, for example, an increase in the energy of the electron results in a corresponding decrease in the cyclotron frequency owing to the increase in relativistic mass. The electron thus experiences a slightly advanced phase of the laser on the next pass through the focus. This implies that ΔW decreases slightly and therefore the electron energy decreases. It is in this manner that the electron phase locks to the laser frequency.

We note that if initially, or at any later time, an energy imbalance exists between the energy absorbed and the energy radiated, then the automatic tracking of the electron to the balance point is not critically damped. Rather, the electron slews to the lock point in a slowly damped oscillatory fashion with a time constant equal to the trap coupled radiative decay time[1] of approximately 1 ms.

To be more quantitative, we write the change of phase of the microwave field (Φ) as

$$\frac{d\Phi(t)}{dt} = \dot{\phi}(t) + \omega_c'$$

where $\dot{\phi}(t)$ is the frequency deviation from the normal "locked" cyclotron frequency ω_c'. We have

$$\dot{\phi}(t) = \Delta\omega(t) = \Delta\omega_{fb}(t) + \Delta\omega_d(t) \qquad (3)$$

where $\Delta\omega_{fb}$ is the is the instantaneous frequency deviation of the electron from ω_c' due to the energy imparted by the laser, and $\Delta\omega_d$ is the frequency deviation due to the radiation decay. Writing $\delta = \pi/2 - \theta$ and assuming we are in the high power limit of the laser then the nominal value of $\delta \rightarrow \pi/2$ so that Eq. (1) becomes $\Delta W \cong W_o\theta$, where $W_o \equiv \sqrt{2}\ eA \exp[-(\pi c S_o/2v\lambda)^2]$. Now $\theta = k\phi$ where k is the division factor v_ℓ/v_c' so that $\Delta W \cong W_o k\phi$.

Noting that

$$\frac{d(\Delta\omega_{fb})}{dt} \cong -\frac{\omega_c'}{E}\left(\frac{dE}{dt}\right)_{fb}$$

where E is the total energy of the electron, then

$$\frac{d(\Delta\omega_{fb})}{dt} = -\omega_\ell\left(\frac{W_o v_c'}{E}\right)\phi \qquad (4)$$

To estimate $\dfrac{d(\Delta\omega_d)}{dt}$ we first note that

$$\frac{d(\Delta\omega_d)}{dt} \cong -\frac{\omega_c'}{E}\left(\frac{dE}{dt}\right)_d$$

An accurate expression for the energy decay due to synchrotron radiation $(dE/dt)_d$ in the absence of coupling structures is given in Ref. 1. Using this expression, we can find the dependence of $d(\Delta\omega_d)/dt$ on the difference frequency ($\Delta\omega$) and obtain

$$\frac{d(\Delta\omega_d)}{dt} \cong -\left(\frac{4e^2\gamma^2\omega_c'^2}{3Ec}\right)\Delta\omega \equiv -\frac{2}{\tau}\Delta\omega \qquad (5)$$

where τ is approximately equal to the damping time at low energy. We will assume that this expression is also valid when τ is decreased by coupling the electron to the trap electrodes. Differentiating Eq. 3 with respect to time and using Eqs. (4) and (5) we obtain

$$\ddot{\phi} + \frac{2}{\tau}\dot{\phi} + (\omega_\phi)^2\phi = 0 \qquad (6)$$

where $\omega_\phi = (\omega_\ell v_c' W_o/E)^{\frac{1}{2}}$. Thus phase oscillations around the nominal "locked" phase occur at frequency ω_ϕ and are damped with approximately the radiation damping time. For a laser power of 200 mW (λ = 3.39 μm) in the above example $W_o \cong 1.8 \times 10^{-13}$ ergs, and $\omega_\phi/2\pi \cong 1.7 \times 10^8$ Hz.

Spread of the electron wave packet

To estimate the restrictions placed on the model by quantum mechanics we start with the uncertainty relation $\Delta\phi\ \Delta n \gtrsim 1$, where $\Delta\phi$ is the uncertainty in phase of the cyclotron orbit of the electron and Δn represents the corresponding uncertainty in energy for the electron. Neglecting the electric field and electron spin, the electron energy is given by

$$E = [(mc^2)^2 + mc^2\hbar\omega_c(n + \tfrac{1}{2})]^{\frac{1}{2}} \qquad (7)$$

We want to build a wave packet which has its phase defined with an accuracy $\Delta\phi \lesssim (2\pi\lambda/10)/(2\pi r_c) \cong 1.3 \times 10^{-4}$ rad. From the uncertainty relation this requires a spread in energy quantum number of $\Delta n \lesssim 0.75 \times 10^4$. From Eq. (7) this corresponds to a range of natural frequencies $\Delta\omega \cong \omega_c'\ \Delta n/n \cong 1.3 \times 10^{-6}\ \omega_c'$. Classically, if we assume that the initial conditions for Eq. 6 are given by these values of $\Delta\phi$ and $\Delta\omega$, then we see that the phase is bound and initially oscillates with amplitude 1.75×10^{-4} rad. Quantum mechanically, it therefore seems likely that the electron will phase lock if the initial wave packet has a limited range of values of n and ϕ. Clearly this treatment is not rigorous and a more careful analysis of the quantum fluctuations must be made.

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